Online Appendix for "Only Here to Help? Bargaining and the Perverse Incentives of International Institutions"

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1 Proof of Proposition 1

Since the institution is fully informed and moves last, simple backward induction finds its move. Because the cost functions are strictly concave and its utility function for reducing the states' costs is weakly concave, the institution's objective function has a unique solution. Call the inputs for that solution α_1^* and α_2^*.

State 2 knows its type when it takes its move and infers that the institution will choose α_2^*. Therefore, by backward induction, a type with cost c_2' accepts if:

$$1 - x > 1 - \alpha_2^* c_2'$$

$$c_2' > \frac{x}{\alpha_2^*}$$

By analogous argument, State 2 rejects if the inequality is flipped. It is indifferent when there is equality, but the equilibrium action taken here is irrelevant because that type has measure zero.

Thus, the strategic tension is in State 1's initial offer. State 1 infers that the institution will choose α_1^* and α_2^*, and it knows how each type of State 2 will act given that they all know α_2^*. It also observes the prior distribution of types $F(c_2)$. Consequently, State 1 must find the optimal demand x that maximizes its expected utility over that distribution and the corresponding actions later in the game. The only possible optimal demands are $x \in [1-\alpha_2^*\underline{c}_2, 1-\alpha_2^*\bar{c}_2]$. This is because all other demands either induce rejection with certainty (making $x = 1 - \alpha_2^*\underline{c}_2$ a profitable deviation) or give more than the minimum amount necessary to induce all types of State 2 to accept (making a slightly larger demand a profitable deviation).

Writing out that utility as a function of x for the remaining interior values gives:

$$F\left(\frac{x}{\alpha_2^*}\right)(-\alpha_1^* c_1) + \left[1 - F\left(\frac{x}{\alpha_2^*}\right)\right](x)$$

where $F\left(\frac{x}{\alpha_2^*}\right)$ is the probability that State 1's demand induces rejection (because State 2's costs are less than $\frac{x}{\alpha_2^*}$) and $\left[1 - F\left(\frac{x}{\alpha_2^*}\right)\right]$ is the probability that State 1's demand is accepted.

Using the product and chain rules, the first order condition of this is:

$$1 - F\left(\frac{x}{\alpha_2^*}\right) - \frac{xf\left(\frac{x}{\alpha_2^*}\right) + \alpha_1^* c_1 f\left(\frac{x}{\alpha_2^*}\right)}{\alpha_2^*} = 0$$

$$\frac{\alpha_2^*}{x + \alpha_1^* c_1} = \frac{f\left(\frac{x}{\alpha_2^*}\right)}{1 - F\left(\frac{x}{\alpha_2^*}\right)} \tag{1}$$

The left-hand side is strictly decreasing in x, while the right-hand side is the distribution's hazard function and is therefore strictly increasing in x. Let x^* be the implicit solution to this function. If x^* is within the interior interval, then it is the unique solution, and State 1 demands that amount. Some portion of the types reject. If x^* does not exist because $\frac{\alpha_2^*}{\alpha_1^* c_1 + \alpha_2^* \underline{c}_2} < \frac{f(\underline{c}_2)}{1 - F(\underline{c}_2)}$ (because c_1 is too large, as Proposition 1 alludes to), then State 1 chooses $x = \alpha_2^* \underline{c}_2$. Note that $\frac{\alpha_2^*}{\alpha_1^* c_1 + \alpha_2^* \underline{c}_2}$ is decreasing in c_1. Thus, for sufficiently large c_1, State 1 selects the offer that State 2 is guaranteed to accept. This completes the proof for Proposition 1. □

2 Proof of Proposition 2

There are two cases to consider: the interior solution and the corner solution. In the corner solution, State 1 makes the safe demand that results in guaranteed acceptance. Slight changes to α_1^* do not alter this. However, sufficient decreases to α_1^* shift the parameters into the interior solution, which features positive probability of bargaining breakdown.

Going further into the interior case requires investigating how the implicit solution to Equation 1 changes as α_1^* decreases. This has the effect of decreasing the denominator on the left-hand side, which increases the left-hand side overall. To compensate, State 1 must increase x to maintain the equality, as this decreases the left-hand side while increasing the right-hand side.

Altering α_1^* has no direct effect on State 2's accept/reject decision. Thus, decreasing α_1^* increases the demand x, which reduces the portion of types of State 2 that accepts. Therefore, the probability of bargaining breakdown increases. □

3 Proof of Proposition 3

There are again two cases to consider: the interior solution and the corner solution. This time, I begin with the interior. The hazard is strictly increasing, and increasing α_2^* *decreases* the input. Thus, the right hand side of Equation 1 is strictly decreasing in α_2^*. Meanwhile, the left hand side is strictly increasing. As such, increasing α_2^* requires State 1 to increase x to maintain the equality. So the optimal demand is increasing in α_2^*.

Let $I(\alpha_2^*)$ be the implicit function that takes α_2^* as the input and outputs State 1's optimal demand. By the above, $I'(\alpha_2^*) > 0$. To see how changing α_2^* affects the probability of breakdown, note that the probability is $F\left(\frac{I(\alpha_2^*)}{\alpha_2^*}\right)$. Consider the derivative with respect to α_2^* by the chain rule:

$$f\left(\frac{I(\alpha_2^*)}{\alpha_2^*}\right)\left(\frac{I(\alpha_2^*)}{\alpha_2^*}\right)'$$

The derivative is increasing if:

$$f\left(\frac{I(\alpha_2^*)}{\alpha_2^*}\right)\left(\frac{I'(\alpha_2^*)\alpha_2^* - I(\alpha_2^*)}{(\alpha_2^*)^2}\right) > 0$$

$$I'(\alpha_2^*)\alpha_2^* > I(\alpha_2^*) \tag{2}$$

This is the condition on the distribution function for the results presented. As such, reducing α_2^* reduces the probability of bargaining breakdown for this class of distribution functions.[1]

Now consider the corner solution. The proof for Proposition 1 shows that the corner solution begins for sufficiently small α_2^*. Further reductions to α_2^* maintain the corner solution and the zero probability of breakdown.

In turn, the only remaining question is how changing α_2^* transitions from the interior to the corner. The transition occurs as α_2^* decreases. In the interior, the probability of breakdown is decreasing as α_2^* decreases. It then becomes static at 0 once in the corner solution. As such, decreasing α_2^* weakly decreases the probability of bargaining breakdown as the proposition claimed. □

[1] Rearranging Equation 2, note that the result also applies locally to any distribution when $\alpha_2^* > \frac{I(\alpha_2^*)}{I'(\alpha_2^*)}$.

4 Proof of Proposition 4

As in previous cases, the institution selects α_1^* and α_2^*. State 2 has complete information and accepts if its draw for p_1 meets $1 - x \geq 1 - p_1 - c_2$, or $x \leq p_1 + c_2$.

State 1 must optimize given its prior beliefs. All types of State 2 accept all x less than or equal to $\underline{p}_1 + \alpha_2^* c_2$. Consequently, the optimal demand must be at least $\underline{p}_1 + \alpha_2^* c_2$. State 1's utility function for x values between $\underline{p}_1 + \alpha_2^* c_2$ and $\bar{p}_1 + \alpha_2^* c_2$ is

$$\int_{\underline{p}_1}^{x-\alpha_2^* c_2} (p_1 - \alpha_1^* c_1) f(p_1) dp_1 + \int_{x-\alpha_2^* c_2}^{\bar{p}_1} x f(p_1) dp_1$$

where the first portion is State 1's expected value for war multiplied by the probability that the demand results in war, while the second portion is State 1's share of the settlement multiplied by the probability that State 2 accepts.

Leibniz's rule and some manipulation produces the first order condition:

$$-f(x - \alpha_2^* c_2)(\alpha_1^* c_1 + \alpha_2^* c_2) + 1 - F(x - \alpha_2^* c_2) = 0$$

$$\frac{1}{\alpha_1^* c_1 + \alpha_2^* c_2} = \frac{f(x - \alpha_2^* c_2)}{1 - F(x - \alpha_2^* c_2)}$$

The left-hand side is constant and the right-hand side is strictly increasing in x. Thus, if x^* solves the equation, it is the unique maximizer, and State 1 chooses that amount in equilibrium. If no such x value exists (because $\frac{1}{\alpha_1^* c_1 + \alpha_2^* c_2} < \frac{f(\underline{p}_1)}{1 - F(\underline{p}_1)}$), State 1 demands $\underline{p}_1 + \alpha_2^* c_2$ instead.[2]

Now to the comparative static. Decreasing α_1^* has a straightforward effect. It decreases the left-hand side's denominator and therefore increases the overall value of the left-hand side. Thus, to maintain the equality, State 1 must choose a larger x value. But the probability of acceptance $F(x - \alpha_2^* c_2)$ is decreasing in x, meaning that this increases the probability of rejection in interior cases. In the corner solution, this can either maintain the zero-probability of war demand or shift the demand into the interior, which permits positive probability of war. Both of these cases comport with Proposition 4's claim.

Decreasing α_2^* has an identical effect, though there is an additional wrinkle: α_2^*

[2]The first order condition rules out all demands greater than $\bar{p}_1 + \alpha_2^* c_2$, as State 1's utility for these demands are identical to demanding $\bar{p}_1 + \alpha_2^* c_2$.

appears in the distribution functions f and F. Fortunately, this is only a superficial problem: α_2^* only shifts where the distribution function starts and finishes, and does not distort it in any other way.[3] Thus, decreasing α_2^* keeps the value for $\frac{f(x-\alpha_2^* c_2)}{1-F(x-\alpha_2^* c_2)}$ for any given x value when pegged to the initial start value. Meanwhile, it strictly decreases the denominator of the left-hand side, leading to an overall increase to the left-hand side's value. From here, the proof is identical to the case of manipulating α_1^*. □

5 Extension: Rivalrous Institutional Effort

Consider the following alternative specification for the institution's utility function. Imagine that the institution has a fixed level of effort $\bar{e}$ that it must exert to alleviate the costs of State 1 and State 2. Further, suppose that the real costs State i suffers is $\frac{c_i}{1+e_i}$, where $e_i \geq 0$ and $e_1 + e_2 = \bar{e}$. Converting this functional form into the language of the original model $\alpha_i = \frac{1}{1+e_i}$.

The interesting question is how the institution's behavior changes as its bias for one state over the other changes. Let $\beta_i \geq 0$ represent the institution's bias for State i. Its overall utility function is then:

$$-\frac{\beta_1 c_1}{1+e_1} - \frac{\beta_2 c_2}{1+e_2}$$

Note that larger values of β_i make the negative effect of that state's cost appear greater to the institution, which is exactly what this parameter is designed to encapsulate.

Substituting $e_2 = \bar{e} - e_1$ and taking the first order condition yields:

$$\frac{\beta_1 c_1}{(1+e_1)^2} = \frac{\beta_2 c_2}{(1+\bar{e}-e_1)^2}$$

The claim from the paper is that increasing bias toward one state increases the effort to that state and decreases effort to the other state. This is easily observed by examining the first order condition. Increasing β_1 increases the overall value of the left hand side of the equation. But the first order condition requires both sides to maintain equality. The only freedom the institution has is to manipulate e_1. Since the right

[3] This is in contrast to when there is uncertainty over State 2's costs, in which case α_2^* compacts the distributions.

hand side is unchanging in β_1, the only way to maintain the inequality is to increase e_1, which causes a decrease in e_2. The argument works analogously for increases to β_2. □

6 Mechanism Design Analysis

Here, I consider a more general version of the problem with uncertainty over the distribution through conflict. Each side remains fully informed of the other's cost. However, Nature now begins by drawing t_i for each player from a commonly known prior distribution. The outcome of a rejected offer $p(t_1, t_2)$ is a function of both of these types, where p is increasing in State 1's type and decreasing in State 2's; that is, a "stronger" type of a state receives a better outcome regardless of the opponent's type.

This is sufficient for the following proposition:

Proposition. *With uncertainty over the distribution of gains, for sufficiently great costs of conflict, there exist efficient, voluntary, and incentive compatible mechanisms. However, for sufficiently effective institutions, all such mechanisms cease to exist.*

The proof is a straightforward corollary to Fey and Ramsay's (2011) Propositions 5 and 6. The relevant comparison is the strategic constraints in all bargaining games with the institution and without. However, because the institution merely reduces costs of both players and this role is common knowledge, the relevant comparison in practice is how the strategic constraints of all bargaining games change as a function of the cost parameters of the states.

Under the conditions of the model, these assumptions meet the requirements of Fey and Ramsay with incomplete information about the relative distribution of power. This unlocks all claims they prove about such games. Their Proposition 6 says that there exists a $\bar{c}$ such that for $c_1 + c_2 \geq \bar{c}$ a mechanism exists that has 0 probability of conflict. Because agreement is certain, the expected inefficiency equals 0. However, by their Proposition 5, if $c_1 + c_2 < \bar{c}$, all mechanisms are inefficient. Thus, an institution in this case must be the cause of that inefficiency. □