En bloc analysis of exhaustive exercise and recovery phases during the cycle ergometer test

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Abstract

Problem Statement: The heart rate (HR) recovery time profile during and after exercise training as a function of load (P) indicates the cardiovascular fitness and health of experimental participants. The HR variability during the recovery phase is closely related to the degree of intensity and duration of the load during exercise training.

Approach: The aim of this study was to propose a mathematic dynamic model that is capable of en bloc analysis of the load effect on the heart rate (HR) recovery time profile. The model is based on a linear differential equation of the exhaustive exercise and 15minute recovery phases of the cycle ergometer test.

Purpose: In total, 14 young, healthy footballers, 20 other sportsmen and 9 sportswomen participated in the test. The starting load was 50W with increasing increments of 20W every minute until maximal exhaustion was reached. The model calculated a new HRR_0 parameter for the period immediately after the end of the exercise phase and the HRR_1 recovery during the first minute after the end of the exercise, according to the model parameters.

Results: The recovery load model parameters was significantly statistically lower for the sportswomen compared with the other sportsmen ($p=0.008$). When the load was increased by 1W at a steady state, the HR was 27.27% higher for the sportswomen compared with the footballers and other sportsmen.

Conclusion: The study proposes a modification of the standard HR evaluation test using a new approach that is capable of estimating the HR recovery from model parameters without using HR measurements during the initial minute after exhaustive exercise. The load and recovery phase were analyzed en bloc using a simple mathematical dynamic model with a minimum number of parameters, as expressed by a linear differential equation of the HR time profile measurements.

Keywords: cardiovascular response; dynamic modeling; exercise; maximal exhaustion

Introduction

The heart rate (HR) recovery time profile during and after load (P) exercise training indicates cardiovascular fitness and the health of participants. The HR variability during the recovery phase is closely related to the degree of intensity and duration of the load in exercise training (Grazzi *et al.* 2005, Vaverka *et al.* 2015; Etxebarria *et al.* 2014). Regardless of the body position before and after the exercise phase, the measured HR fall might not reach the baseline values after 5 minutes (Barak *et al.* 2011) or 15 minutes (Barak *et al.* 2010) of recovery after short-term submaximal cycling on an ergometer with an intensity of 80% of individual peak HR values.

When limiting individual maximal power output in the exercise phase to 70%, the HR gradually decreases during the recovery phase and returns to the pre-exercise levels after 30 minutes of recovery (Javorka *et al.* 2002). The kinetics of the HR during exercise and recovery has been estimated by various models analyzing both exercise and recovery phases separately, and the HR recovery alone (Arduini *et al.* 2011, Baig *et al.* 2010, Stirling *et al.* 2008, Su *et al.* 2010).

Our study was aimed to apply a system approach to en bloc analysis of the load and recovery phases, using a mathematical model with the minimum number of parameters, and in the form of a differential equation relating load effect on the HR time profile. The proposed dynamic model assumes that the load (P) exercise phase is finished at the point of maximal exhaustion and that in the recovery phase the HR does not return to its baseline within 15 minutes after completion of the exercise in the cycle ergometer test.

Methods

Subjects

A group of thirty four sportsmen and nine sportswomen (aged 19 to 26 years) with no health problems volunteered to participate in this study. The subjects' basic characteristics are summarized in Table 1. All the sportsmen actively participated in football ($n = 14$) and other kinds of sport, i.e. floorball ($n = 5$), basketball ($n = 2$), volleyball ($n = 2$), baseball ($n = 1$), shooting ($n = 4$), strength training ($n = 3$), cycling ($n = 2$) and swimming ($n = 1$). The sportswomen participated only recreationally in aerobics ($n = 5$), strength training ($n = 2$) and shooting ($n = 2$) and in mandatory physical education of 2 hours per week. The procedures and protocols for the study were performed in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments, or with comparable ethical standards.

Table 1. Subjects' basic characteristics and their statistical comparison. Data are means $\pm$ standard deviation (SD).

#	Sportsmen ($n = 34$)		Sportswomen ($n = 9$)	One-way ANOVA test (p value)		
	Footballers ($n = 14$)	Other sportsmen ($n = 20$)		Footballers/Other sportsmen	Footballers/ Sportswomen	Other sportsmen/ Sportswomen
Age (years)	21.14 $\pm$ 1.4	21.45 $\pm$ 1.6	20.67 $\pm$ 1.2	NS	NS	NS
Height (m)	1.81 $\pm$ 0.07	1.81 $\pm$ 0.05	1.71 $\pm$ 0.06	NS	0.002	«0.001
Weight (kg)	81.41 $\pm$ 12.8	80.56 $\pm$ 10.4	62.97 $\pm$ 7.4	NS	0.002	0.001
BMI (kg.m ⁻²)	25.76 $\pm$ 3.9	24.23 $\pm$ 2.6	21.44 $\pm$ 1.9	NS	0.008	NS
Body fat (%)	13.23 $\pm$ 6.3	11.68 $\pm$ 4.93	18.51 $\pm$ 3.2	NS	NS	0.008
Body fat (kg)	11.37 $\pm$ 7.2	9.73 $\pm$ 4.9	11.81 $\pm$ 3.1	NS	NS	NS
MET	11.8 $\pm$ 2.1	12.29 $\pm$ 2.09	10.78 $\pm$ 3.3	NS	NS	NS

BMI - Body Mass Index; MET - metabolic equivalent; NS - not significant; $p < 0.05$ was considered significant; n - number of participants

Cycle ergometer testing protocol

Prior to exercise testing, each participant was informed of the procedure for the exercise load test. The exercise tests to evaluate the HR and maximal oxygen uptake, VO_{2max} , were carried out in the sports centre of the Faculty of Mechanical Engineering of the Slovak University of Technology, Bratislava, using a cycle ergometer (TUNTURI, Almere, Norway, software T-ware 2). A protocol with a maximal load (P_{max}) was applied; participants were presented with increasing loads up to maximal effort and the maximal cardiovascular capacity of the volunteer. All of the participants were given progressive loads on an electrically controlled cycle ergometer starting at 50 W. Subsequently, the load was increased in steps of 20 W every minute without a break until the maximal exhaustion of the participant. Each participant was asked to maintain a pedalling frequency of 60 revolutions per minute. Objective indicators included the maximal heart rate (HR_{max}) over measured time t_{max} according to the age and gender of the participant.

The protocol included measurements of blood pressure and HR both during the exercise and the inactive recovery phase, recorded five times at intervals of three minutes after completing the exercise on the ergometer. The inactive recovery phase consisted of the participants sitting at rest for 15 minutes with measurements of the aforementioned physiological values at the listed time intervals. The listed variables were measured by a digital cuff blood pressure monitor (OMRON 705IT, Kyoto, Japan). The body profile consisted of BMI and fat content values. The percentage of fat and its conversion to kilograms was carried out based on Bioelectrical Impedance Analysis (BIA), by a digital monitor (OMRON BF300, Kyoto, Japan). The maximal oxygen uptake (VO_{2max}) was calculated in $ml.kg^{-1}.min^{-1}$ by an indirect method based on the heart rate, with a subsequent conversion to assign the value to the metabolic equivalent (MET) scale.

Statistical analysis

The significance of the differences between the groups of footballers, other sportsmen, and the sportswomen in terms of the measured, model and derived parameters was estimated via multiple one-way analysis of variance

(ANOVA) with Bonferroni correction. The value $p < 0.05$ was considered statistically significant. The correlations between the model and the derived parameters were evaluated by Pearson correlation coefficients.

Mathematical modeling

A reasonable dynamic model of heart rate $HR(t)$ time profile is proposed in Eq. (1):

$$HR(t) = HR_b + \overline{HR}(t) \quad (1)$$

where HR_b is the basal heart rate and $\overline{HR}(t)$ time profile represents the analytical solution of differential Eq. (2) with $P(t)$ as an input to the model:

$$\overline{HR}(t) + T \frac{d\overline{HR}(t)}{dt} = G \cdot P(t) \quad (2)$$

where t is time, T is the time constant, G is the gain of the analyzed system and $P(t)$ is the time profile of the load shown in Fig. 1(A) as a ramp function.

In the steady state Eq. (3) is valid:

$$G = \frac{HR_{ss}}{P_{ss}} \quad (3)$$

where G expresses the HR increase at a steady state when the load is increased at SS by 1 Watt. A schematic illustration of the heart rate time profile $HR(t)$ during the exhaustive exercise and recovery phases on the cycle ergometer is shown in Fig. 1(B).

Fig. 1. Scheme of (A) the ramp function of load time profile $P(t)$ and (B) heart rate time profile $HR(t)$ from the cycle ergometer test before and after exercise.

t - time; t_{max} - maximal time to exhaustion at P_{max} ; P_{max} - maximal load during exercise; P_r - equivalent recovery load after exercise; HR_b - basal heart rate; HR_{max} - maximal heart rate during exercise; HR_r - recovery heart rate after exercise; tangent line 0 - derivation of $HR(t)$ with convergence $t \rightarrow t_{max}$ from the right at the point of exercise termination expressed by Eq. (A.12) in the Appendix; tangent line 1 - derivation of descending recovery phase $HR(t)$ in $t_{max}+1$ expressed by Eq. (A.11) in the Appendix.

For the estimation of the model parameters of the vector $\lambda = \{T, G, P_r\}$, the methods of Monte Carlo (Lee 1968, Manno 1999) and Gauss-Newton (Heath 2002) included in the CCSS (Computer Controlled Sequential Simulation) method (Dedík *et al.* 2009) were used.

The structure of the model expressed by Eqs (1) and (2) was determined on the basis of the minimal value of Akaike information criterion (Akaike 1976).

Results

The overview of mean values of the measured, model and the derived parameters were related to the HR and other significantly different parameters of footballers, sportsmen and sportswomen during the load exercise and recovery phases of the exhaustive cycle ergometer test. These, together with their statistical comparison according to a one-way ANOVA test with Bonferroni correction, are shown in Table 2.

Table 2. Summary of the statistical comparison of measured, model and derived parameters of HR (A), and other parameters (B). Data are means $\pm$ standard deviation (*SD*).

(A)						
#	Sportsmen (<i>n</i> = 34)		Sportswomen (<i>n</i> = 9)	One-way Anova test (<i>p</i> value)		
	Footballers (<i>n</i> = 14)	Other sportsmen (<i>n</i> = 20)		Footballers/Other sportsmen	Footballers/Sportswomen	Other sportsmen/Sportswomen
HR _b (beats.min ⁻¹)	78.43 $\pm$ 14.6	72.95 $\pm$ 10	77.89 $\pm$ 15.1	NS	NS	NS
HR _{max} (beats.min ⁻¹)	155.84 $\pm$ 22.7	157.04 $\pm$ 19.4	153.59 $\pm$ 15.4	NS	NS	NS
HR _r (beats.min ⁻¹)	103.92 $\pm$ 13.1	102.36 $\pm$ 11.2	103.38 $\pm$ 12.3	NS	NS	NS
Δ HR (beats.min ⁻¹)	25.50 $\pm$ 11.6	29.41 $\pm$ 6.8	25.49 $\pm$ 14.3	NS	NS	NS
HRR ₀ (beats.min ⁻¹)	28.12 $\pm$ 17.3	35.43 $\pm$ 17.7	29.99 $\pm$ 10.8	NS	NS	NS
HRR ₁ (beats.min ⁻¹)	20.69 $\pm$ 9.2	25.99 $\pm$ 11	22.68 $\pm$ 7.3	NS	NS	NS
(B)						
#	Sportsmen (<i>n</i> = 34)		Sportswomen (<i>n</i> = 9)	One-way Anova test (<i>p</i> value)		
	Footballers (<i>n</i> = 14)	Other sportsmen (<i>n</i> = 20)		Footballers/Other sportsmen	Footballers/Sportswomen	Other sportsmen/Sportswomen
t _{max} (min)	11.19 $\pm$ 2.3	10.72 $\pm$ 2.2	8.12 $\pm$ 2.7	NS	0.02	0.03
P _{max} (W)	253.6 $\pm$ 35.8	252 $\pm$ 33.7	181.11 $\pm$ 36.6	NS	«0.001	«0.001
P _r (W)	63.85 $\pm$ 23.3	76.71 $\pm$ 20.1	47.39 $\pm$ 24.7	NS	NS	0.008
T (min)	2.35 $\pm$ 1.1	1.76 $\pm$ 0.5	1.76 $\pm$ 0.5	NS	NS	NS
G (beats.min ⁻¹ .W ⁻¹)	0.4 $\pm$ 0.2	0.4 $\pm$ 0.1	0.55 $\pm$ 0.2	NS	NS	NS

HR_b - basal heart rate; HR_{max} - maximal heart rate during exercise; HR_r - recovery heart rate after exercise; Δ HR, HRR₀ and HRR₁ - derived parameters of heart rate expressed by Eq. (A.4), Eq. (A.13) and Eq. (A.9) in the Appendix; t_{max} - maximal time to exhaustion at P_{max}; P_{max} - maximal load during exercise; P_r - equivalent recovery load after exercise; T - time constant; G - gain; NS - not significant; *p* < 0.05 was considered significant; *n* - number of subjects.

As shown in Table 2(A), from the viewpoint of the ANOVA test with Bonferroni correction, all the HR parameters were statistically non-significant between the three tested sport groups (*p* > 0.05), probably due to a high standard deviation. Table 2(B) shows our statistically significant parameters t_{max}, P_{max} and P_r in the three tested sports groups. The measured parameter t_{max}, *i.e.* the maximal time to exhaustion at P_{max}, was the statistically significant lower in sportswomen compared with footballers (*p* = 0.02) and other sportsmen (*p* = 0.03). Similarly, the measured parameter P_{max}, *i.e.* the maximal load, was statistically significantly lower in sportswomen compared with footballers (*p* < 0.001) and other sportsmen (*p* < 0.001). The model parameter P_r, *i.e.* the recovery load, was statistically significantly lower in sportswomen compared with the other sportsmen (*p* = 0.008).

The results of the regression analysis of the relationships between the derived parameters HRR₁ and HRR₀ for two sportsmen's and one sportswomen's groups are illustrated in Fig. 2.

Fig. 2. Relationship between the derived parameters HRR_1 and HRR_0 .

(A) circles - footballers, full circles - footballers with extraordinary health responses during the test; (B) squares - other sportsmen; (C) triangles - sportswomen. Line - regression line.

The dependence between the derived parameters in the tested groups is expressed by a regression line; in footballers the slope is 1.206 ± 0.022 and the intercept zero, and the Pearson correlation coefficient is $r = 0.981$. Three footballers with exceptional health responses during the ergometer test, (one footballer with a high HR_b (114 beats.min⁻¹) and two footballers with a rapid HR decrease in the third minute after maximal exhaustive cycling), are marked by full circles (Fig. 2(A)). In relation to the other sportsmen's group, the slope is 5.819 ± 1.99 and the intercept is 1.587; $r = 0.982$ (Fig. 2(B)). With regard to the sportswomen, the slope is 1.333 ± 0.034 and the intercept is zero; $r = 0.978$ (Fig. 2(C)). The results of the regression analysis of the relationship between the derived parameters in the form $(HR_{max} - HR_b)/P_{max}$, and the model parameter - gain G - for two sportsmen's and one sportswomen's groups are illustrated in Fig. 3.

Fig. 3. Relationship between $(HR_{max} - HR_b)/P_{max}$ and gain G.

(A) circles - footballers; (B) squares - other sportsmen; (C) triangles - sportswomen. Line - regression line.

The dependence between the parameters in the presented groups is expressed by a regression line. In footballers, the slope is 1.521 ± 0.093 and the intercept is -0.0735 ± 0.031 ; $r = 0.978$ (Fig. 3(A)). In relation to the other sportsmen's group, the slope is 1.182 ± 0.018 and the intercept is zero; $r = 0.96$ (Fig. 3(B)). With regard to the sportswomen, the slope is 1.273 ± 0.034 and the intercept is zero; $r = 0.976$ (Fig. 3(C)).

Discussion

Our work presents a proposal for a new system approach to the analysis of the HR time profile during the load phase of exhaustive exercise and the following recovery phase in two groups of sportsmen (footballers and others), and one group of sportswomen exercising on a cycle ergometer. The load and recovery phase are analyzed en bloc by a simple mathematical dynamic model, with a minimum number of parameters, expressed by a linear differential equation of the HR time profile measurements.

As shown in Table 2(A), our parameters HR_r and ΔHR in active footballers (103.92 ± 13.1 beats.min⁻¹ and 25.50 ± 11.6 beats.min⁻¹, respectively) had statistically higher values ($p < 0.001$) compared with the study of Desalegn and Verma (2002), where the HR_r was 92.41 ± 4.87 beats.min⁻¹ after 15 minutes of recovery phase. The difference between HR_b and HR_r , i.e. ΔHR , in 149 well-trained players at Punjabi University on the exhaustive cycle ergometer test after the 15 minutes of recovery was 18.43 ± 5.5 beats.min⁻¹ (Desalegn and Verma 2002). The rate of HR recovery to baseline values is markedly dependent on the efficiency and the fitness of the sportsmen. It confirms Du *et al.* (2005), who observed a faster HR recovery in female marathon runners compared to the untrained control volunteers after maximal running exercise on a treadmill. In a study using similar body positioning before and after the exercise phase, Barak *et al.* (2010) observed a markedly lower difference between HR_b and HR_r during the last 5 minutes of the 15 minute recovery in a cycle ergometer test, i.e. $\Delta HR = 9$ beats.min⁻¹ in healthy young students. While the load exercise phase in our study was maintained to the point of maximal exhaustion, the participants in the study of Barak *et al.* (2010) underwent five minutes of submaximal cycling with an intensity of only 80% of their individual peak HR values. This different load at the end of the exercise phase could lead to a different convergence of recovery HR to baseline values in comparison to our study. As shown in Table 2(B), our results of P_{max} in sportswomen (181.11 ± 36.6 W) demonstrate a statistically non-significant difference ($p > 0.05$) compared with the results in the study of Hofmann *et al.* (1994), where, in 16 healthy female physical education students who did not participate in any regular sports and after the exhausting cycle ergometer test (40 W initial work load and subsequent 10 W.min⁻¹ load increase), P_{max} was 193.1 ± 23.6 W. The variation of P_{max} could be explained by the different sport activity, age, vital and physical capacity, and the anthropometric parameters of the participants, etc. Bovens *et al.* (1993) observed that in 2,038 men active in endurance sports, and 898 women over 40 years of age, after the cycle ergometer with a P increase 50 W/2.5min up to a HR of between 140 and 150 beats.min⁻¹, and following a P increase of 25 W/2.5min until exhaustion, the P_{max} that could be maintained for 2.5 min, was 254.3 ± 46.1 W in the men and 161.7 ± 28.2 W in the women. In comparison with our results, a statistically non-significant difference ($p > 0.05$) was shown compared to our tested active footballers (253.6 ± 35.8 W), and a statistically significant difference ($p = 0.04$) compared to our sportswomen (181.11 ± 36.6 W).

Based on the differential equations of the HR time profile, our proposed model enables us to estimate a new parameter P_r , the equivalent load in the recovery phase after exercise training. According to our results, Akaike's information criterion for a model with $P_r > 0$ and $HR_r > HR_b$ was lower than in the model with $P_r = 0$ and $HR_r = HR_b$. The parameter HR recovery, HRR_1 , is calculated by the standard procedure directly from the measured values of the maximal HR at time t_{max} , i.e. $HR(t_{max})$, and from the HR at 1 minute after the end of the exhaustive exercise, i.e. $HR(t_{max}+1)$. However, it is significantly influenced by measurement error in these times. The stated measurement error is removed by Eq. (A.10), which allows the calculation of HRR_1 from the model parameters as the derivation of a model HR response in the time $t_{max}+1$. Our proposed parameter HRR_0 gives a derivation of $HR(t)$ immediately after the termination of the exercise phase and can be calculated only from the model parameters (see Eq. (A.13)), and does not require explicit measurement in the period of 1 min after maximal load. Parameter α (see Eq. (A.14)) shows that HRR_0 always has a higher value than HRR_1 .

From the viewpoint of the Pearson correlation coefficient, the close correlation between the derived parameters HRR_1 and HRR_0 in footballers ($r = 0.981$), other sportsmen ($r = 0.982$) and sportswomen ($r = 0.978$) (Fig. 2) shows that the time constant T represents a significant model parameter for en bloc analysis of the HR time profile during the load and recovery phases of this test, as shown by Eqs. (A.10) - (A.13).

A close correlation between derived parameters, expressed as the ratio $(HR_{max} - HR_b)/P_{max}$, and the gain G modeled by the proposed mathematical model, with $r = 0.978$ in footballers, $r = 0.96$ in other sportsmen and $r = 0.976$ in sportswomen, was observed (Fig. 3).

Conclusions

A new system approach capable of the en bloc evaluation of heart rate time profile during the exhausting exercise and recovery phases of a cycle ergometer test of footballers, other sportsmen and sportswomen is proposed. This method of analysis is based on a mathematical model of the relationship between load and heart

rate time profiles according to a linear differential equation. Our mathematical model allows the estimation of new parameters, such as the apparent limit of the recovery heart rate and recovery load observed up to 15 min after exhausting exercise, and the time constant. Based on the equations in the Appendix, the model approach is capable of estimating the generally used parameter recovery heart rate from the model parameters without using measurements of heart rate at 1 min after exhausting exercise, and modifies the standard evaluation of the heart rate time profile for the proposed system approach in the cycle ergometer test.

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Conflict of interest

There is no conflict of interest.

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Appendix

The analytical solution of Eq. (1) and differential Eq. (2) for a load P time profile shown in Fig. 1 is expressed by Eq. (A.1):

$$P(t) = \begin{cases} \frac{P_{\max}}{t_{\max}} t; \text{ for } t \in [0, t_{\max}] , \\ P_r; \text{ for } t > t_{\max} \end{cases} \quad (\text{A.1})$$

the heart rate HR time profile for the exhausting exercise phase is expressed by Eq. (A.2):

$$HR(t) = H_b + G(t_{\max} - T(1 - e^{-\frac{t}{T}})) \frac{P_{\max}}{t_{\max}} t; \text{ for } t \in [0, t_{\max}] , \quad (\text{A.2})$$

the HR time profile in the recovery phase after exercise is expressed by Eq. (A.3):

$$HR(t) = HR_r + e^{\frac{t_{\max}-t}{T}} (HR_{\max} - HR_r); \text{ for } t > t_{\max} , \quad (\text{A.3})$$

where HR_b is basal heart rate, t and $t_{\max}$ are time and maximal time of exercise, respectively, $P_{\max}$ is maximal load during exercise, G is gain (expressing the increase in HR when the load is increased at a steady state by 1 Watt), and T is the time constant.

The parameter ΔHR , defined as the difference between HR_b and HR_r , is expressed by Eq. (A.4):

$$\Delta HR = HR_r - HR_b = G \cdot P_r \quad (\text{A.4})$$

The apparent limit of recovery heart rate and the equivalent recovery load - HR_r and P_r , respectively, observed until 15 min after exhausting exercise, are expressed by Eq. (A.5):

$$HR_r = HR_b + G \cdot P_r \quad (\text{A.5})$$

and Eq. (A.6):

$$P_r = \frac{\Delta HR}{G} \quad (\text{A.6})$$

The maximal HR at time $t_{\max}$ during exercise is expressed by Eq. (A.7):

$$HR_{\max} = HR(t_{\max}) = HR_b + G(t_{\max} - T(1 - e^{-\frac{t_{\max}}{T}})) P_{\max} \quad (\text{A.7})$$

HR at 1 min after exhausting exercise is expressed by Eq. (A.8):

$$HR(t_{\max} + 1) = HR_r + e^{\frac{-1}{T}} (HR_{\max} - HR_r) \quad (\text{A.8})$$

HR recovery according to previous studies (Duarte *et al.* 2014, Heffernan *et al.* 2007, Jehn *et al.* 2011, Legramante *et al.* 2007) is expressed by Eq. (A.9):

$$HRR_1 = HR(t_{\max}) - HR(t_{\max} + 1) \quad (\text{A.9})$$

HR recovery expressing the decline of HR frequency at 1 min after exercise, according to our mathematical model is formulated by Eq. (A.10):

$$HRR_1 = (HR_{\max} - HR_r)(1 - e^{\frac{-1}{T}}) \quad (\text{A.10})$$

The derivation of the descending recovery phase $HR(t)$ is expressed by Eq. (A.11):

$$\frac{dHR(t)}{dt} = -\frac{1}{T} e^{\frac{t_{\max}-t}{T}} (HR_{\max} - HR_r); \text{ for } t > t_{\max} \quad (\text{A.11})$$

The derivation of $HR(t)$ with convergence $t \rightarrow t_{\max}$ from the right at the termination of exercise is expressed by Eq. (A.12):

$$\left. \frac{dHR(t)}{dt} \right|_{t=t_{\max}}^+ = -\frac{1}{T} (HR_{\max} - HR_r) \quad (\text{A.12})$$

Consequently, the parameter HRR_0 expressing the decline of HR frequency at 0 min after exercise, established by taking the derivation of $HR(t)$ from the right, is expressed by Eq. (A.13):

$$HRR_0 = \left| \frac{dHR(t)}{dt} \right|_{t=t_{\max}}^+ = \frac{1}{T} (HR_{\max} - HR_r) \quad (\text{A.13})$$

For the comparison of HRR_0 and HRR_1 according to parameter α , Eq. (A.14) that $HRR_0 > HRR_1$ is valid because

$$\alpha = \frac{HRR_0}{HRR_1} = \frac{\frac{1}{T}}{1 - e^{\frac{-1}{T}}} > 1 \quad (\text{A.14})$$