ROBUST STABILITY INVESTIGATION OF LQG AND H_∞ CONTROLLERS FOR ELECTROHYDRAULIC STEERING CONTROL SYSTEM

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Abstract. The paper presents a comparison of the robust stability of two embedded controllers intended for application in electrohydraulic steering systems – Linear-Quadratic Gaussian (LQG) controller and H_∞ controller. Both controllers are synthesized on the basis of state-space system model obtained by identification procedure. To investigate robust stability of the closed-loop system 30% parametric uncertainty is introduced into B matrix of the model which describes the observed dead-band of the electrohydraulic proportional spool valve. The closed-loop system is represented as $M - \Delta$ interconnection which allows calculation of its structured singular value (μ) with both controllers. The H_∞ controller achieves smaller singular value than the LQG controller which asserts that loop stability in presence of bounded variations in the model characteristics is increased in the case of H_∞ controller. The performance of both controllers is experimentally verified on laboratory test bench.

Keywords: robust stability, LQG, H_∞ , electrohydraulic steering.

1. INTRODUCTION

Steering system of mobile machines operates on a hydrostatic principle and does not include any rigid mechanical linkage from the operator steering wheel to the steering cylinder. The energy required for the steering is transmitted by the pressure and flow rate quantities of a compressible hydraulic fluid. In addition to steering function the hydraulic system within mobile machines supports also other essential machine functions such as working operations,

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DOI: 10.7546/EngSci.LVII.20.04.02

movement, brakes, etc. A Hydrostatic Steering Unit (HSU) supplies with the flow rate through the steering cylinder which is proportional of the operator command. The command is classically formed by a steering wheel. Other alternatives for command generation are joystick panels or remote path tracking control. Introduction of an electrohydraulic control module into steering system allows such functions to be incorporated within HSU. This ability to manipulate hydraulic variables by electronic signals stimulates development of embedded systems targeted for hydraulic applications. Electrohydraulic steering systems are generally composed of conventional HSU, flow rate amplification valve (Load-Sensing), pilot operated electro-proportional valve, steering servo cylinder and sensors [1]. The ultimate purpose of embedded electronics in hydraulic devices is the improvement of operational performance, diagnostic capabilities and safety. Since the hydraulic machines are designed with respect to maximizing efficient energy utilization, some constructive elements are characterized with nonlinearities. The uncertainty caused by such elements can impair closed-loop stability and performance. Embedded control algorithms have to be carefully examined with respect to uncertainties before deployment into actual machine.

Fig. 1. Hydraulic scheme of power steering system with digital valves

Figure 1 presents simplified hydraulic scheme of the power steering system with digital valve actuation together with its control system.

The main purpose of this paper is to be done a comparative analysis between the robust stability of LQG and H_∞ digital controllers designed for electrohydraulic power steering system with hydrostatic transmission. The mathematical model used for the controllers design is third order state-space representation obtained with identification procedure from experimental data. The model is disturbed by two uncertain scalar elements which account for 30% variability of respective model parameters. The robust stability of the uncertain closed loop is assessed by calculation of bounds for the structured singular value μ .

The robustness of a system means that it retains certain properties, regardless of the variations in the parameters of its internal elements within the permissible limits.

2. UNCERTAIN SYSTEM MODEL

Authors have developed a laboratory hydraulic test equipment for EHSU type OSPEC200 LSRM, taking into account current technical specifications from the manufacturer [1]. Figure 2 shows hydraulic diagram of the test bench system, described in detail in [2].

A digital control system has been developed in which an electronic joystick is configured for an input device. It consists of an electronic joystick, a digital LQG regulator supplying control voltage to the PVE module, which in turn controls the PVE built-in electrohydraulic proportional valve. The electrohydraulic proportional valve (pos. 6, Fig. 2) determines the direction of movement of the executive hydraulic cylinder (pos. 12, Fig. 2) by feeding a working fluid to one of both chambers. The plant output is a cylinder piston position that is measured through a sensor. The measured sensor signal is used in feedback and is compared with the reference signal.

Digital control system is based on a controller type MC012-022 and electronic joystick type JS6000 (Danfoss platform for mobile applications), by which operated module with two-way, two-position valves connected in parallel – PVE.

In the programming environment (PLUS + 1 Guide) [3] controller MC012-022 is created and a program that calculates parameters of control signal. The experimental system (Fig. 1) consists of EHSU type OSPE 200 and symmetric servo cylinder connected by pipelines through L and R.

and correlation tests. The details on identification can be found in [5]. The estimated state space model has the form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + K_v v(k) \\ y(k) = Cx(k) + v(k) \end{cases}, \quad (1)$$

where $x(k) = (x_1, x_2, x_3)^T$ is a vector with the state variables, $u(k)$ is the input signal, $y(k) = (y_{press}, y_{pos})^T$ is the output vector signal which components are the pressure drop and the piston position and A, B, C, K_v are the matrices:

$$A = \begin{pmatrix} 0.8769 & -0.3987 & 0.3986 \\ 0 & 0 & 1 \\ -0.1666 & -0.5099 & 1.509 \end{pmatrix}, \quad B = \begin{pmatrix} -0.0043 \\ 0.0011 \\ 0.0021 \end{pmatrix}, \quad (2)$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad K_v = \begin{pmatrix} 0.1112 & -0.06214 \\ -0.09525 & 1.55 \\ -0.2003 & 1.897 \end{pmatrix}$$

Due to the hysteresis of the main directional valve which is related to the positive overlap of its edges the uncertainty of 30% is introduced in matrix B of the model (1) as:

$$\tilde{B} = B + \begin{pmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & \delta_3 \end{pmatrix} |0.3B|, \quad (3)$$

where $\delta_1, \delta_2, \delta_3 \in [-1, 1]$ are normalized uncertain scalar variables independent of each other. Thus, the model (1) takes a form:

$$\begin{cases} x(k+1) = Ax(k) + \tilde{B}u(k) + K_v v(k) \\ y(k) = Cx(k) + v(k) \end{cases}. \quad (4)$$

3. LQG AND H_∞ CONTROLLERS DESIGN

The structure scheme of control system with designed LQG controller and Kalman filter is shown in Fig. 3. To ensure sufficiently well reference tracking an LQG controller with integral action is designed. The design of controller and Kalman filter is done on the basis of deterministic part of estimated model (1) according to procedure described in details in [4]. According to the results presented in [4] the optimal controller gain is:

$$K = [K_c \ -K_i], \quad (5)$$

where K_c is a matrix of the state proportional gains and K_i is the integral gain designed according to [4] with the following weighting matrices for the state feedback:

$$Q = \text{diag} (7 \times 10^4 \ 10^5 I_{2 \times 2} \ 0), \quad R = 5 \times 10^3, \quad (6)$$

where the first state corresponds to the output error integrator which extends the state-space description. The Kalman filter is designed with MATLAB® [5, 6] with respect to the disturbance matrix K_v from the identification model. The obtained model (1) is used also for H_∞ controller [6, 7] design according to closed-loop diagram presented in Figure 4, which is corresponding to the S/KS sensitivity criterion.

Fig. 3. LQR with Kalman filtering closed-loop scheme

Fig. 4. Structure for H_∞ controller design

The design of controller is done according to procedure described in details in [5]. The transfer functions of weighting filters are:

$$W_p = \text{diag} \left(0.1 \ 0.005 \ \frac{0.8(0.1s + 1)}{0.5s + 1} \right), \quad W_u = 0.08. \quad (7)$$

The obtained value for parameter γ is 0.669, which means that the performance criteria are satisfied. Figure 5 compares the sensitivity of the output to load disturbances of both the closed-loop systems. The comparison shows that they have similar performance, but the H_∞ controller achieves wider bandwidth and 10 dB additional attenuation in the low frequency range. Figure 6 compares the sensitivity of the control signal to measurement noise which is higher for the H_∞ controller. There the LQG controller performs better because it prevents high frequency noises from disturbing the actuator. So, the H_∞ controller would be appropriate only if the applied sensors generate a low amount of noise [8].

Fig. 5. Output sensitivity of the closed-loop with LQG and H_∞ controllers

Fig. 6. Sensitivity of the control action to noises of the closed-loop with LQG and H_∞ controllers

4. ROBUST STABILITY INVESTIGATION OF SYSTEM WITH LQG AND H_∞ CONTROLLERS

To investigate the effect of the uncertainty on the closed-loop system with the LQG controller or H_∞ controller the following expressions are used:

$$\begin{cases} \begin{pmatrix} x(k+1) \\ x_c(k+1) \end{pmatrix} = \begin{pmatrix} A_c & \tilde{B}C_c \\ -B_cC & A_c \end{pmatrix} \begin{pmatrix} x(k) \\ x_c(k) \end{pmatrix} + B_{LQG} \begin{pmatrix} 0 \\ r(k) \end{pmatrix} \\ y(k) = (C \ 0) \begin{pmatrix} x(k) \\ x_c(k) \end{pmatrix} \end{cases}, \quad (8)$$

where A_c , B_c , C_c is the state-space representation of the designed controller and x_c is its internal state vector. There it can be observed that the uncertain elements from the open-loop matrix $\tilde{B}$ are expressed also in the matrix A of the closed loop system. Hence the introduced uncertainty can influence the stability of the loop [9].

To investigate the robust stability of the closed-loop system the description (8) is represented by upper LFT shown in Fig. 7.

The transfer matrix M is calculated from state space description:

$$\begin{cases} \begin{pmatrix} x(k+1) \\ x_{LQG}(k+1) \end{pmatrix} = \begin{pmatrix} A & BC_{LQG} \\ -B_{LQG}C & A_{LQG} \end{pmatrix} \begin{pmatrix} x(k) \\ x_{LQG}(k) \end{pmatrix} + \begin{pmatrix} I \\ 0 \end{pmatrix} u_{\Delta} \\ y_{\Delta} = (0 \ 0.3|B|C_{LQG}) \begin{pmatrix} x(k) \\ x_{LQG}(k) \end{pmatrix} \end{cases}, \quad (9)$$

where $u_{\Delta} = \Delta y_{\Delta}$ and $\Delta = \text{diag}(\delta_1, \delta_2, \delta_3)$.

It is well known that the system presented in Fig. 6 will achieve robust stability for all $\|\Delta\| < 1$ if and only if [10, 11]:

$$\sup_{\omega \in R} \mu_{\Delta}[M(j\omega)] < 1, \quad (10)$$

where $\mu_{\Delta}[M(j\omega)]$ is the structured singular value.

The results for upper bound of structured singular value of closed-loop systems with LQG and H_{∞} controllers are presented in Fig. 8. As can be seen the supremum of the structured singular value of system with H_{∞} controller is about 0.06, while one for the system with LQG controller is about 0.3. Thus, both the systems are robust stable but the system with H_{∞} controller can tolerate approximately 16 times larger uncertainty than one presented in model (4) and the system with LQG controller can tolerate approximately 3 times larger uncertainty than prescribed one.

In Figure 9 the transient responses of the closed-loop systems with LQG and H_{∞} controllers with respect to the position are shown.

They are calculated for 50 various values of uncertain parameters generated by Monte Carlo simulation. It is seen that the settling time of system with H_{∞} controller is approximately 4 s and all transient responses are without overshooting. The settling time of system with LQG controller is 9 s and some of transient responses have small overshooting. The performance of

Fig. 7. $M - \Delta$ structure of closed-loop system

Fig. 8. Robust stability of uncertainty closed-loop system with LQG and H_{∞} controllers

Fig. 9. Step response of the closed-loop position system with LQG and H_{∞} controllers

closed-loop system based on the H_∞ controller is better than one of control system with LQG controller.

5. EXPERIMENTAL RESULTS

The designed H_∞ and LQG controllers are implemented in target micro-controller MC012-022 by STL language and tested into the test bench, Fig. 10. For this aim controller interconnection is represented in equivalent single matrix vector formulas:

$$u(k+1) = \begin{pmatrix} A_c & B_c \\ C_c & D_c \end{pmatrix} e(k). \quad (11)$$

The elements of the controller matrices are represented in a fixed-point format with scaling of 1000.

In Figure 11 the comparison between results for the cylinder piston position obtained from systems with H_∞ and LQG controllers is presented. The analogous results for control signal are presented in Fig. 12.

Measured reactions of the cylinder piston are aperiodic with setting time of approximately 3.5 s (H_∞ controller) and 15 s (LQG controller) for 1/3 of the piston stroke. This is acceptable for low speed steering of heavy duty machines. There is zero steady state error for system with LQG controller and sufficiently small steady state error for system with H_∞ controller. The quality of the transition processes is maintained when cylinder piston moving

Fig. 10. Photo of the laboratory test-bench with its main functional components

Fig. 11. Transient response of closed-loop systems

Fig. 12. Control signals of closed-loop systems

in both directions. The control signal approaches its maximum value during the transition process, indicating that the output reacts with the maximum possible performance boost.

6. CONCLUSION

The article presents a comparative analysis of the robust stability property of embedded systems for control of electrohydraulic power steering based on LQG and H_∞ controllers. Robust stability was investigated using the identified model of the steering system with input multiplicative uncertainty, which takes into account the deviations of the parameters that characterize the way in which the control signal acts on the state of the model. The designed controllers are implemented in 32-bit microcontroller. For this aim appropriate real-time software is developed. A number of experiments have been executed, which confirm the quality of the electrohydraulic power steering control system. The presented experimental results confirm that the control system achieves the prescribed performance.

ACKNOWLEDGEMENTS

This work was supported by the Bulgarian Ministry of Education and Science under the National Research Program E+ “Low carbon energy for transport and live”, approved by DCM No 577/17.08.2018.

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Received November 19, 2020