DETERMINATION OF THE CRITICAL LOADING AREAS ON THE DISCRETE SPHERICAL DOMES USING INFLUENCE SURFACES

ANITA HANDRULEVA

*USEA “Lyuben Karavelov”, Faculty of Construction,
175, Suhodolska St., 1373 Sofia, Bulgaria,
e-mail: anita_handruleva@abv.bg*

Abstract. In this article, the influence surfaces of discrete spherical domes have been investigated. Using both the discrete and Finite Element Method (FEM) for frames to generate five different models of domes, being the objects of my study, is discussed herein. Computational models differ in the type of grid configuration. Selected frames are in the meridian, parallel and diagonal directions, as for each frame an influence surface modelled by exerted vertical load to the spherical surface has been built. Research and graphical representations have been processed by using software based on the FEM, and employing static and kinematic methods. For comparison, the influence surfaces introduced the following parameters: absolute density, compressive density and tensile density. Based on the aforesaid parameters the following correlated factors have been determined: coefficient of compressive activation and coefficient of tensile activation. Results are presented in graphical and tabular form.

Keywords: spherical domes, discrete element modelling, FEM.

1. INTRODUCTION

Influence Surface (IS) for effort (support reaction, bending moment, shear and normal forces) is called three-dimensional graph showing the change of this magnitude as a function of the position of the moving concentrated force with a single value and constant direction, applied to the “road” surface, see [1–5].

In flat lattice systems it is a set of lines of influence, built on separate orthogonal sections with equal intervals on the surface of the investigated object. Many difficulties in the study of domes exist. The distance between sections

DOI: 10.7546/EngSci.LVII.20.03.02

decreases in the height of the meridian, and the geometric area that simulates IS is with a double curvature and has got to find a “readable” form for their presentation. The question is: “How to be an IS displayed? Either as a projection on a horizontal plane that illustrates insufficiently readable the change of IS in the areas around the supporting ring (base supports), or via a cartographic projection by unfolding of the spherical surface onto a vertical plane?” In this case the distances in-between the meridians at/around the top of the dome appear to be greatly exaggerated, thence the surface gets deformed. A third option is a perspective presentation of the dome in a selected view, as the ordinates of the influence surface are applied linear-vertically. This option is adopted for further submission. The manual calculation of the ordinates of this graph for space systems by means of method of displacements and method of forces is almost impossible due to the high static indeterminateness, see [6–10]. The calculation of the ordinates of an IS for a particular exerted effort by using advanced software programs on Finite Element Method (FEM) (SAP2000, ANSYS, etc.) has been really improved, which in turn facilitates the entire object- or element-based modelling process, see [11–13]. As well-known as it is from Structural Mechanics, there are three methods for the construction of lines of influence that can be applied to build the IS: static, kinematic and a combined one.

The altering of the studied parameters in value and sign has been approached by applying the static method when changing the location of the single moving force in the defined road sections. For each position of the simulated force exerted, an equation of equilibrium is recorded, and (searched) an assumed response effort is calculated, etc.

The kinematic method is based on the principle of possible displacement, whereat the equations drawn for the work of all forces acting on the system, previously converted into a mechanism by removing the link, or supporting the effort, are recorded.

Influence Surfaces have been used to study the domes in the stage of limit equilibrium (loss of stability) and in the study of loss of bearing capacity of the elements.

2. MAIN OBJECTIVES AND TASKS RELATED TO THE STUDY OF THE INFLUENCE SURFACES

- To outline areas for obtaining extreme (compressive and tensile) values for efforts in the bars of the dome with different morphology of the grid configuration under the action of vertical load to the spherical surface;

- To identify areas of IS in which the load does not affect significantly on a rod test effort, which would simplify the procedure for seeking extreme efforts;
- To clarify the role of symmetric, asymmetric and local load on the state of the rods.

3. DESCRIPTION OF SELECTED COMPUTATIONAL MODELS

As objects to study, the surfaces of influence in lattice structures are considered as representatives of the five domes. The domes are conditionally named K.1÷K.5, respectively, as shown in Fig. 1.

Fig. 1. Selected representatives of the single layer spherical domes

To generate the models the following fixed parameters have been adapted, see Fig. 2:

Fig. 2. Geometrical characteristics of numerical models

- the diameter of the base $D = 30$ m;
- a diameter of the ring at the top $r_k = 1.65$ m;
- central semi angle 50° with the corresponding height 7 m;
- a cyclic angle at the base $\theta = 15^\circ$;
- number of parallels separating meridian lines (inner rings) $n = 7$;
- rigid connection between the elements of the grid structure;
- the domes have radially movable supports;
- one-dimensional finite elements of a Frame type were employed to generate the computational models of single-layer discrete spherical domes;
- For the structural elements, cross-sections of pipe profiles have been chosen, namely: $\varnothing 83 \times 4$ of $A = 9.93 \text{ cm}^2$ area, a moment of inertia $I = 77.64 \text{ cm}^4$ and a radius of inertia $i = 2.8 \text{ cm}$. The profiles are made of steel class S235JRH following EN 10219-2 with a calculated resistance of steel 235 MPa.

For all elements of the studied domes, tubular profiles with equal geometric characteristics of the cross-section have been adapted.

Bars in the meridian and parallel direction, and diagonals are selected. For every one of them, an IS of vertical load to the spherical surface is built. In meridian direction, elements are conventionally denoted by R1÷R8, in the ring (parallel) – with N1÷N7, and diagonals – with D1÷D8.

The study and graphic imaging have been done with a SAP.2000 (version 15) software based on the FEM using both the static and kinematic method.

4. COMPARATIVE ANALYSIS OF THE RESULTS

Figures 3 through 7 show the spatial images of the influence surface of the axial force for a selected element.

Fig. 3. Influence surface for normal effort built upon a static method for dome K.1

Fig. 4. Influence surface for normal effort built upon a static method for dome K.2

Fig. 5. Influence surface for normal effort, built upon a static method for dome K.3

Fig. 6. Influence surface for normal effort built upon a static method for dome K.4

Fig. 7. Influence surface for normal effort built upon a static method for dome K.5

In the Figures from 8 to 11 another approach has been used to illustrate the influence surface for a normal force by the application of the kinematic method. In this visualization, the areas of tensile and compressive activation in the studied element are clearly outlined/articulated.

The kinematic method is based on the principle of possible displacements. In this method, equations for the work of the forces acting on the system have been recorded. It was turned into a mechanism by removing the connection R , supporting the effort. As well known, following the principle of possible displacements $F \cdot \bar{\delta}_f + R \cdot \bar{\delta}_R = 0$.

When $F = 1$ we obtain:

$$R = -\frac{\bar{\delta}_f}{\bar{\delta}_R}, \quad (1)$$

where $\bar{\delta}_f$ is the displacement of the application point of moving force, decomposed in its direction, i.e. combination of decomposed displacements of all crawled by force F points on the way; $\bar{\delta}_R$ is the mutual displacement in the points of application of force R , decomposed in its direction.

At possible displacement $\bar{\delta}_R = -1$ the surface of decomposed displacements would be identical to the surface of the influence of the effort: “ R ” = “ $\bar{\delta}_f$ ”.

In other words, the influence surface is defined as elastic surface – diagram of the displacements of the points of the nodes of the structure in the negative single mutual displacement or rotation in the direction of the effort.

Fig. 8. Influence surface for normal effort in element from dome K.1 built employing the kinematic method: (a) N1; (b) N2

Fig. 9. Influence surface for normal effort in element from dome K.1 built using the kinematic method: (a) N3; (b) N4

Fig. 10. Influence surface for normal effort in element from dome K.1 built using the kinematic method: (a) N5; (b) N6

Fig. 11. Influence surface for normal effort in element N7 from dome K.1, built using the kinematic method

After processing the results (in the case of ordinates η_i of the influence surface) for the evaluation of IS for efforts in the elements from different computational models following new evaluative-quantitative indicators have been introduced. See Tables 1÷13:

- Absolute density $D_a = \sum |\eta_i|$;
- Compression density $D_c = \sum (-\eta_i)$;

Table 1. Evaluative-quantitative indicators and correlated coefficients [*...] of elements N1÷N8 from dome K.1

Dome K.1	Element N1	Element N2	Element N3	Element N4	Element N5	Element N6	Element N7	Element N8
D_a	10.0409	10.7765	13.6341	17.1060	22.3683	28.2988	23.4068	55.2918
D_c	3.3686	4.4642	5.6479	7.4039	9.5027	11.8863	15.3172	47.2442
D_t	6.6723	6.3123	7.9862	9.7021	12.8656	16.4126	8.0896	8.0476
K_c	0.3355	0.4143	0.4142	0.4328	0.4248	0.4200	0.6544	0.8545
K_t	0.6645	0.5857	0.5858	0.5672	0.5752	0.5800	0.3456	0.1455

Table 2. Evaluative-quantitative indicators and correlated coefficients [*...] of elements R1÷R8 from dome K.1

Dome K.1	Element R1	Element R2	Element R3	Element R4	Element R5	Element R6	Element R7	Element R8
D_a	17.2045	17.2399	17.4409	18.4791	20.1780	22.3439	23.9732	19.3688
D_c	14.0631	14.0416	14.0337	14.5614	15.4723	16.8190	18.1104	14.8186
D_t	3.1414	3.1983	3.4072	3.9177	4.7057	5.5249	5.8628	4.5502
K_c	0.8174	0.8145	0.8046	0.7880	0.7668	0.7527	0.7554	0.7651
K_t	0.1826	0.1855	0.1954	0.2120	0.2332	0.2473	0.2446	0.2349

Table 3. Evaluative-quantitative indicators and correlated coefficients [*...] of elements N1÷N8 from dome K.2

Dome K.2	Element N1	Element N2	Element N3	Element N4	Element N5	Element N6	Element N7	Element N8
D_a	13.9494	15.2844	17.5825	20.3577	25.1654	32.6626	28.0829	50.7764
D_c	5.4331	6.5678	7.5960	8.9871	10.8107	14.1039	17.5656	44.3627
D_t	8.5164	8.7166	9.9865	11.3706	14.3547	18.6237	10.5173	6.4137
K_c	0.3895	0.4297	0.4320	0.4415	0.4296	0.4318	0.6255	0.8737
K_t	0.6105	0.5703	0.5680	0.5585	0.5704	0.5702	0.3745	0.1263

Table 4. Evaluative-quantitative indicators and correlated coefficients [*...] of elements R1÷R8 from dome K.2

Dome K.2	Element R1	Element R2	Element R3	Element R4	Element R5	Element R6	Element R7	Element R8
D_a	10.9198	12.1115	14.6621	16.2071	16.9858	17.4847	19.1164	16.3471
D_c	10.8441	10.8724	12.3520	13.1906	13.7025	14.2277	15.4805	14.0620
D_t	0.0757	1.2391	2.3100	3.0164	3.2833	3.2570	3.6358	2.2851
K_c	0.9931	0.8977	0.8424	0.8139	0.8067	0.8137	0.8098	0.8602
K_t	0.0069	0.1023	0.1576	0.1861	0.1933	0.1863	0.1902	0.1398

Table 5. Evaluative-quantitative indicators and correlated coefficients [*...] of elements D1÷D8 from dome K.2

Dome K.2	Element D1	Element D2	Element D3	Element D4	Element D5	Element D6	Element D7	Element D8
D_a	12.1226	13.0328	14.6688	16.7523	19.1577	22.3267	24.4963	26.0024
D_c	5.4328	6.8346	7.3951	5.4357	9.6013	11.2302	13.1852	13.9198
D_t	6.6997	6.1981	7.2736	8.3166	9.5565	11.0965	11.3111	12.0826
K_c	0.4482	0.5244	0.5041	0.3245	0.5012	0.5030	0.5383	0.5353
K_t	0.5527	0.4756	0.4959	0.4964	0.4988	0.4970	0.4617	0.4647

Table 6. Evaluative-quantitative indicators and correlated coefficients [*...] of elements N1÷N8 from dome K.3

Dome K.3	Element N1	Element N2	Element N3	Element N4	Element N5	Element N6	Element N7	Element N8
D_a	13.7855	15.1941	17.5804	20.3818	25.0989	33.4057	24.6515	49.8503
D_c	5.1478	6.7076	7.6527	9.0062	10.7678	14.6477	16.0271	43.9952
D_t	8.6377	8.4865	9.9278	11.3756	14.3311	18.7580	8.6243	5.8551
K_c	0.3734	0.4415	0.4353	0.4419	0.4290	0.4385	0.6501	0.8825
K_t	0.6266	0.5585	0.5647	0.5581	0.5710	0.5615	0.3499	0.1175

Table 7. Evaluative-quantitative indicators and correlated coefficients [*...] of elements R1÷R8 from dome K.3

Dome K.3	Element R1	Element R2	Element R3	Element R4	Element R5	Element R6	Element R7	Element R8
D_a	13.5967	15.7131	17.5510	19.4723	21.7993	24.2707	23.2571	22.0172
D_c	12.2456	13.2787	14.0800	15.0363	16.2600	17.7614	17.5627	15.4309
D_t	1.3512	2.4344	3.4710	4.4360	5.5393	6.5093	5.6944	6.5863
K_c	0.9006	0.8451	0.8022	0.7722	0.7459	0.7318	0.7552	0.7009
K_t	0.0994	0.1549	0.1978	0.2278	0.2541	0.2682	0.2448	0.2991

Table 8. Evaluative-quantitative indicators and correlated coefficients [*...] of elements D1÷D8 from dome K.3

Dome K.3	Element D1	Element D2	Element D3	Element D4	Element D5	Element D6	Element D7	Element D8
D_a	11.7705	12.9569	14.6666	16.6465	19.0829	21.9382	21.3920	20.6520
D_c	5.8715	6.4817	7.3353	8.3249	9.5509	10.9880	10.8271	10.9372
D_t	5.8990	6.4752	7.3313	8.3216	9.5320	10.9502	10.5649	9.7148
K_c	0.4988	0.5003	0.5001	0.5001	0.5005	0.5009	0.5061	0.5296
K_t	0.5012	0.4997	0.4999	0.4999	0.4995	0.4991	0.4939	0.4704

Table 9. Evaluative-quantitative indicators and correlated coefficients [*...] of elements M1÷M8 from dome K.4

Dome K.4	Element M1	Element M2	Element M3	Element M4	Element M5	Element M6	Element M7	Element M8
D_a	15.0321	15.9238	17.7497	20.1321	25.0068	33.7270	24.8276	49.5665
D_c	3.6188	5.0722	6.1157	7.5480	9.6862	13.9915	15.5381	43.8742
D_t	11.4133	10.8517	11.6340	12.5841	15.3205	19.7355	9.2895	5.6923
K_c	0.2407	0.3185	0.3446	0.3749	0.3873	0.4148	0.6258	0.8852
K_t	0.7593	0.6815	0.6554	0.6251	0.6127	0.5852	0.3742	0.1148

Table 10. Evaluative-quantitative indicators and correlated coefficients [*...] of elements P1÷P8 from dome K.4

Dome K.4	Element P1	Element P2	Element P3	Element P4	Element P5	Element P6	Element P7	Element P8
D_a	12.4551	14.1464	15.4310	16.8028	18.6490	21.3389	21.4709	20.9030
D_c	9.8312	10.5013	10.9141	11.4367	12.2578	13.6313	13.8274	13.0120
D_t	2.6239	3.6451	4.5169	5.3661	6.3913	7.7076	7.6435	7.8910
K_c	0.7893	0.7423	0.7073	0.6806	0.6573	0.6388	0.6440	0.6225
K_t	0.2107	0.2577	0.2927	0.3194	0.3427	0.3612	0.3560	0.3775

Table 11. Evaluative-quantitative indicators and correlated coefficients [*...] of elements N1÷N8 from dome K.5

Dome K.5	Element N1	Element N2	Element N3	Element N4	Element N5	Element N6	Element N7	Element N8
D_a	5.946866	9.7083	12.5719	16.1382	22.3229	31.0855	26.4422	50.9588
D_c	2.7507	3.2219	4.1863	5.7485	7.9730	11.6697	15.4092	44.5902
D_t	3.1962	6.4864	8.3856	10.3898	14.3499	19.4158	11.0330	6.3686
K_c	0.4625	0.3319	0.3330	0.3562	0.3572	0.3754	0.5827	0.8750
K_t	0.5375	0.6681	0.6670	0.6438	0.6428	0.6246	0.4173	0.1250

Table 12. Evaluative-quantitative indicators and correlated coefficients [*...] of elements R1÷R8 from dome K.5

Dome K.5	Element R1	Element R2	Element R3	Element R4	Element R5	Element R6	Element R7	Element R8
D_a	17.6589	14.8407	16.5226	16.6548	15.7584	13.2865	8.6187	4.2734
D_c	16.8194	12.1017	12.9806	12.6911	11.7694	9.9772	6.7113	3.5235
D_t	0.8395	2.7390	3.5420	3.9637	3.9889	3.3093	1.9074	0.7498
K_c	0.9525	0.8154	0.7856	0.7620	0.7469	0.7509	0.7787	0.8245
K_t	0.0475	0.1846	0.2144	0.2380	0.2531	0.2491	0.2213	0.1755

Table 13. Evaluative-quantitative indicators and correlated coefficients [*...] of elements D1÷D8 from dome K.5

Dome K.5	Element D1	Element D2	Element D3	Element D4	Element D5	Element D6	Element D7	Element D8
D_a	9.4330	7.1655	7.8426	8.7974	10.0477	11.8891	14.0644	15.4755
D_c	2.1349	4.2997	4.3997	5.1010	6.0052	7.3187	9.0521	9.7271
D_t	7.2981	2.8658	3.4429	3.6964	4.0424	4.5705	5.0123	5.7485
K_c	0.2263	0.6001	0.5610	0.5798	0.5977	0.6156	0.6436	0.6285
K_t	0.7737	0.3999	0.4390	0.4202	0.4023	0.3844	0.3564	0.3715

- Tensile density $D_t = \sum (\eta_i)$.

Based on these indicators, new correlated coefficients [*of participation] were derived: Coefficient of compressive activation $K_c = D_c/D_a$ and coefficient of tensile activation $K_t = D_t/D_a$.

5. CONCLUSION

The surfaces of influence give a clear idea of the contribution of the load to the final value of effort in the studied element. It is rather easy for the zones of loading to be determined in order to obtain limit value of a normal effort in the selected elements of the structural grid. The constructed IS has shown that the closest to the investigated element area accounts for the largest contribution to the normal effort in the studied element. When the points of application of the forces are far away from the element, attenuation of the influence may be observed. This effect is spatial and depends on the stiffness of the grid structure. Evidently, as IS is more concentrated in the nearby region around an element (with fast “attenuation”), the dome system is more likely to manifest a greater rigidity. New coefficients of compressive

and tensile activation clearly indicate what part of the loading area of the dome leads to compression or tension in the element. It is easy to follow that for the element R.5 of dome K.1, 77% of the loading area leads to compressive activation and only 23% to tensile activation. In other words, when there is uniformly distributed symmetric load, the zones of tensile activation will reduce the normal force in the element by 23%. Similar conclusions can be deduced for each element based on the new coefficients.

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Received August 13, 2020