MULTIPLICATIVE INDICES OF CARBON GRAPHITE
 t -LEVELS

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Abstract

The molecular topological descriptors are the numerical invariants of a molecular graph and are useful to predict their bioactivity and physiochemical properties. Within this article special attention is focused in the study of chemical graph of carbon graphite and to compute the degree based multiplicative topological indices including multiplicative first and second, hyper first and second and general first and second Zagreb index, multiplicative product and sum connectivity index, multiplicative ABC index, multiplicative GA index and general multiplicative GA index of carbon graphite. Moreover, the exact results of these indices are derived for chemical graph of $CG(m, n)$ t -levels

Key words: molecular descriptor, carbon graphite, multiplicative topological indices

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1. Introduction. The chemical compounds can be modelled by using the mathematical tools of graph theory. The molecular structure of any chemical graph has a lot of chemical and physical information. In order to study this information, we deal with molecular descriptors of a graph. It plays a vital role in the field of chemical sciences. The topological index is a numerical value which has the property to have some information about chemical structure. The role of topological descriptors is very useful in the study of drug designs, pharmaceuticals, medical, and other areas of chemical sciences.

A variety of chemical compounds whether organic or inorganic has enormous applications in different fields and in real life use. Chemical graph theory is a

very interesting field of mathematical chemistry. The multiplicative indices can be used in the study analysis of QSPR/QSAR and play a prominent role in analyzing the boiling point and melting point for medicinal drugs, chemical structures and mathematical chemistry [10]. GAO and FARAHANI computed degree based indices of chemical structures by using edge divided method [8]. GAO et al. [7,8] studied chemical structures in drugs and some drug structures and computed the forgotten topological indices. BAIG et al. [1] computed the topological descriptors of the chemical graphs of carbon graphite and crystal cubic carbon structures and also illustrated their graphical representation. The Zagreb multiplicative indices of derived graphs like line graph, subdivision graph, vertex-semitotal graph, edge-semitotal graph, total graph and paraline graph are computed in [2]. The multiplicative Zagreb indices of an infinite class of titania nanotubes $\text{TiO}_2[m, n]$ is computed in [15]. GAO et al. computed multiplicative indices of certain chemical structures like nanotube, nanostar and polyomino chain [10].

Some other applications are given in [5,6,9,11,16,18] and the references therein, which is helpful for the importance of mathematical chemistry. The forgotten topological index is calculated of several widely used chemical structures which often appear in drug molecular graphs and in various classes of drugs [7,8].

Carbon is an essential element for human life. It is useful in the formation of proteins, carbohydrates and nucleic acids. It is vital for the growth of plants in the form of carbon dioxide. It is in the form of carbonates and bicarbonates in oceans, as a good conductor of electricity and as limestone in rocks. Carbon is capable of forming a lot of allotropes because of its valency. The famous allotropes of carbon are graphite, diamond, bucky ball (buckminsterfullerenene), etc. The applications of different allotropes of carbon specially graphite, diamond and bucky balls are discussed in detail in [17]. The structure of different allotropes of carbon and carbon nanotubes are depicted in Fig. 1.

Let $G = (V, E)$ be a connected molecular graph, where V is the vertex set (atoms) and E is the edge set (chemical bond) of G . The degree $\deg(v)$ of v is the number of edges of G incident with v . The distance $d(u, v)$ of a graph G is defined as the shortest length between u and v .

Topological indices are of many kinds. They depend on the degree, distance and eccentricity of all the vertices of a molecular graph. In this paper, we deal with some multiplicative degree based topological indices. The first and second multiplicative Zagreb indices of a graph G are introduced by TODESHINE et al. in [19] and defined as:

$$II_1(G) = \prod_{u \in V(G)} d_G(u)^2$$

$$II_2(G) = \prod_{uv \in E(G)} d_G(u) \times d_G(v).$$

The important mathematical applications and properties of these indices are dis-

Fig. 1. (a) Diamond (b) Graphite (c) Lonsdaleite (d) C_{60} (buckminsterfullerene) (e) C_{540} Fullerite (f) C_{70} (g) Amorphous carbon (h) Single-walled carbon nanotube

cussed in [3, 5, 12, 16, 20].

In [4], ELIASI et al. proposed a new multiplicative version of the first Zagreb index as:

$$II_1^*(G) = \prod_{uv \in E(G)} d_G(u) + d_G(v).$$

The first and second multiplicative hyper Zagreb indices of a graph G are introduced by KULLI in [13] and defined respectively as:

$$HII_1(G) = \prod_{uv \in E(G)} \left(d_G(u) + d_G(v) \right)^2$$

$$HII_2(G) = \prod_{uv \in E(G)} \left(d_G(u) \times d_G(v) \right)^2.$$

The general first and second multiplicative Zagreb indices of a graph G are introduced by KULLI [12] and defined respectively as:

$$MZ_1^\alpha(G) = \prod_{uv \in E(G)} \left(d_G(u) + d_G(v) \right)^\alpha$$

$$MZ_2^\alpha(G) = \prod_{uv \in E(G)} \left(d_G(u) \times d_G(v) \right)^\alpha.$$

In 2016, KULLI [14] introduced multiplicative connectivity indices, the multiplicative product connectivity index, multiplicative sum connectivity index, multiplicative atom bond connectivity index, multiplicative geometric arithmetic index and also general multiplicative geometric arithmetic index of a graph as follows:

The multiplicative product connectivity index of a graph G is defined as:

$$\chi II(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) \times d_G(v)}}.$$

The multiplicative sum connectivity index of a graph G is defined as:

$$XII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}.$$

The multiplicative atom bond connectivity index of a graph G is defined as:

$$ABCII(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u) \times d_G(v)}}.$$

The multiplicative geometric arithmetic index of a graph G is defined as:

$$GAII(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d_G(u) \times d_G(v)}}{d_G(u) + d_G(v)}.$$

The general multiplicative geometric arithmetic index of a graph G is defined as:

$$GA^\alpha II(G) = \prod_{uv \in E(G)} \left(\frac{2\sqrt{d_G(u) \times d_G(v)}}{d_G(u) + d_G(v)} \right)^\alpha.$$

2. Multiplicative indices of carbon graphite. The main aim of this section is to calculate the degree based multiplicative indices of the chemical graph of carbon graphite for t levels. These indices include multiplicative first and second Zagreb index, multiplicative hyper first and second Zagreb index, general multiplicative first and second Zagreb index, multiplicative product connectivity index, multiplicative sum connectivity index, multiplicative ABC index, multiplicative GA index and general multiplicative geometric arithmetic index for the chemical graph of copper graphite $CG(m, n)$ for t -levels. A topological index can give us chemical information that is related to the chemical graph.

2.1. Carbon graphite $CG(m, n)$. In this subsection, we discuss and study the molecular structure of carbon graphite $CG(m, n)$. The structure of carbon graphite consists of hexagons in the form of layers one after the other and between these layers there is a weak bond. The molecular graph of carbon graphite $CG(m, n)$ for t -levels is depicted in Fig. 2.

The cardinality of vertices and edges in $CG(m, n)$ are $2t(m + n + mn)$ and $4mnt + 3mt + 2nt - mn - m - t$, respectively.

The m is the number of rows and n is the number of columns in each layer. m is taken as n copies of hexagons in a row and n is taken as m copies of hexagons in column.

In $CG(m, n)$, the 2 degree vertices are $2(m + nt + 1)$, the 3 degree vertices are $2mn + 2mt - 2m + 2t - 4$ and 4 degree vertices are $2(mn - 1)(t - 1)$. The edge set of $CG(m, n)$ is divided into six partitions based on the degree of end vertices.

Fig. 2. Carbon graphite $CG(m, n)$ graph for t -levels

Fig. 3. Carbon graphite graph for $(3, 4)$ and $t = 3$

T a b l e 1

Edge partition of $CG(m, n)$ based on degree sum of end vertices of each edge

(d_u, d_v)	frequency
(2, 2)	4
(2, 3)	$4(n + t - 1)$
(2, 4)	$4(nt + m - n - t)$
(3, 3)	$4m + 4t - 10$
(3, 4)	$6mn + 6mt - 14m - 4n - 6t + 12$
(4, 4)	$(4mn - 3m - 2n + 1)t - 7mn + 5m + 4n - 2$

The first edge partition contains 4 edges uv , where $\deg(u) = \deg(v) = 2$. The second edge partition contains $4(n + t - 1)$ edges uv , where $\deg(u) = 2$ and $\deg(v) = 3$. The third edge partition contains $4(m - n - t + nt)$ edges uv , where $\deg(u) = 2$ and $\deg(v) = 4$. The fourth edge partition contains $4m + 4t - 10$ edges uv , where $\deg(u) = \deg(v) = 3$. The fifth edge partition contains $6mn + 6mt - 14m - 4n - 6t + 12$ edges uv , where $\deg(u) = 3$ and $\deg(v) = 4$. The sixth edge partition contains $(4mn - 3m - 2n + 1)t - 7mn + 5m + 4n - 2$ edges uv , where $\deg(u) = \deg(v) = 4$. Table 1 shows such an edge partition of $CG(m, n)$ for $m, n > 1$ and $t \geq 3$. The molecular graph of carbon graphite graph with $(m, n) = (3, 4)$ for $t = 3$ is depicted in Fig. 3. Here, $m = 3$ is taken as 4 copies of hexagons in a row and $n = 4$ is taken as 3 copies of hexagons in column in each layer.

2.2. Results for the carbon graphite. In this subsection, the results are stated in the form of theorems and their proofs. In the following theorem, we compute multiplicative first Zagreb index, a new version of first multiplicative Zagreb index and second multiplicative Zagreb index of copper graphite $CG(m, n)$ t -levels.

Theorem 2.1. *Consider the graph of $G \cong CG(m, n)$ with $m, n > 1$ and $t \geq 3$, then its multiplicative first, a new version first and second Zagreb index are equal to*

$$\begin{aligned}
 II_1(G) &= \left(\frac{2^{12}}{3^8}\right) \left(\frac{2^4}{3^4}\right)^m \left(\frac{3^4}{2^8}\right)^{mn} \left(\frac{2^{8mn+4n-8}}{3^{-6(m+1)}}\right)^t \\
 II_1^*(G) &= \left(\frac{7^{12}}{(2^8)(3^{10})(5^4)}\right) \left(\frac{(2^8)(5^4)}{(3^4)(7^4)}\right)^n \left(\frac{(2^{23})(3^8)}{7^{14}}\right)^m \left(\frac{7^6}{2^{21}}\right)^{mn} \left((2^{12mn-9m-2n+3})(3^{4n})\right. \\
 &\quad \left.(5^4)(7^{6(m-1)})\right)^t \\
 II_2(G) &= \left(\frac{2^{20}}{3^{12}}\right) \left(\frac{2^4}{3^6}\right)^m \left(\frac{3^6}{2^{16}}\right)^{mn} \left(\frac{2^{16mn+4n-16}}{3^{-6(m+1)}}\right)^t.
 \end{aligned}$$

Proof. Let chemical graph $G \cong CG(m, n)$ with $m, n > 1$ and $t \geq 3$. The above result can be proved by using Table 1 in the following computation. The multiplicative first Zagreb index is computed as below:

$$II_1(G) = \prod_{u \in V(G)} d_G(u)^2$$

$$II_1(G) = (2^2)^{2(m+nt+1)} \times (3^2)^{2mn+2mt-2m+2t-4} \times (4^2)^{2(mn-1)(t-1)}$$

$$II_1(G) = \left(\frac{2^{12}}{3^8}\right) \left(\frac{2^4}{3^4}\right)^m \left(\frac{3^4}{2^8}\right)^{mn} \left(\frac{2^{8mn+4n-8}}{3^{-6(m+1)}}\right)^t.$$

The new version of multiplicative first Zagreb index is computed as below:

$$II_1^*(G) = \prod_{uv \in E(G)} d_G(u) + d_G(v)$$

$$II_1^*(G) = (2+2)^4 \times (2+3)^{4(n+t-1)} \times (2+4)^{4(m-n-t+nt)} \times (3+3)^{4m+4t-10} \\ \times (3+4)^{6mn+6mt-14m-4n-6t+12} \times (4+4)^{(4mn-3m-2n+1)t-7mn+5m+4n-2}$$

$$II_1^*(G) = \left(\frac{7^{12}}{(2^8)(3^{10})(5^4)}\right) \left(\frac{(2^8)(5^4)}{(3^4)(7^4)}\right)^n \left(\frac{(2^{23})(3^8)}{7^{14}}\right)^m \left(\frac{7^6}{2^{21}}\right)^{mn} \left((2^{12mn-9m-2n+3})(3^{4n})\right. \\ \left.(5^4)(7^{6(m-1)})\right)^t.$$

The multiplicative second Zagreb index is computed as below:

$$II_2(G) = \prod_{uv \in E(G)} d_G(u) \times d_G(v)$$

$$II_2(G) = (2 \times 2)^4 \times (2 \times 3)^{4(n+t-1)} \times (2 \times 4)^{4(m-n-t+nt)} \times (3 \times 3)^{4m+4t-10} \\ \times (3 \times 4)^{6mn+6mt-14m-4n-6t+12} \times (4 \times 4)^{(4mn-3m-2n+1)t-7mn+5m+4n-2}$$

$$II_2(G) = \left(\frac{2^{20}}{3^{12}}\right) \left(\frac{2^4}{3^6}\right)^m \left(\frac{3^6}{2^{16}}\right)^{mn} \left(\frac{2^{16mn+4n-16}}{3^{-6(m+1)}}\right)^t. \quad \square$$

A close formula of multiplicative first general Zagreb index and multiplicative second general Zagreb index of copper graphite $CG(m, n)$ is computed in the following theorem.

Theorem 2.2. Consider the graph of $G \cong CG(m, n)$ with $m, n > 1$ and $t \geq 3$, then its general first and second multiplicative Zagreb index is equal to

$$MZ_1^\alpha(G) = \left(\frac{7^{12\alpha}}{(2^{8\alpha})(3^{10\alpha})(5^{4\alpha})}\right) \left(\frac{(2^{8\alpha})(5^{4\alpha})}{(3^{4\alpha})(7^{4\alpha})}\right)^n \left(\frac{(2^{23\alpha})(3^{8\alpha})}{7^{14\alpha}}\right)^m \left(\frac{7^{6\alpha}}{2^{21\alpha}}\right)^{mn} \\ \left((2^{12mn-9m-2n+3})(3^{4n})(5^4)(7^{6(m-1)})\right)^{t\alpha}$$

$$MZ_2^\alpha(G) = \left(\frac{2^{20\alpha}}{3^{12\alpha}}\right) \left(\frac{2^{4\alpha}}{3^{6\alpha}}\right)^m \left(\frac{3^{6\alpha}}{2^{16\alpha}}\right)^{mn} \left(\frac{2^{16mn+4n-16}}{3^{-6(m+1)}}\right)^{t\alpha}.$$

Proof. Let chemical graph $G \cong CG(m, n)$ with $m, n > 1$ and $t \geq 3$. The above result can be proved by using Table 1 in the following computation. The multiplicative general first Zagreb index is computed as below:

$$MZ_1^\alpha(G) = \prod_{uv \in E(G)} \left(d_G(u) + d_G(v)\right)^\alpha$$

$$MZ_1^\alpha(G) = (2+2)^{4\alpha} \times (2+3)^{4\alpha(n+t-1)} \times (2+4)^{4\alpha(m-n-t+nt)} \times (3+3)^{(4m+4t-10)\alpha} \times \\ (3+4)^{(6mn+6mt-14m-4n-6t+12)\alpha} \times (4+4)^{((4mn-3m-2n+1)t-7mn+5m+4n-2)\alpha}$$

$$MZ_1^\alpha(G) = \left(\frac{7^{12\alpha}}{(2^{8\alpha})(3^{10\alpha})(5^{4\alpha})}\right) \left(\frac{(2^{8\alpha})(5^{4\alpha})}{(3^{4\alpha})(7^{4\alpha})}\right)^n \left(\frac{(2^{23\alpha})(3^{8\alpha})}{7^{14\alpha}}\right)^m \left(\frac{7^{6\alpha}}{2^{21\alpha}}\right)^{mn}$$

$$\left((2^{12mn-9m-2n+3})(3^{4n})(5^4)(7^{6(m-1)})\right)^{t\alpha}.$$

The multiplicative general second Zagreb index is computed as below:

$$MZ_2^\alpha(G) = \prod_{uv \in E(G)} \left(d_G(u) \times d_G(v)\right)^\alpha$$

$$II_2(G) = (2 \times 2)^{4\alpha} \times (2 \times 3)^{4\alpha(n+t-1)} \times (2 \times 4)^{4\alpha(m-n-t+nt)} \times (3 \times 3)^{(4m+4t-10)\alpha} \times (3 \times 4)^{(6mn+6mt-14m-4n-6t+12)\alpha} \times (4 \times 4)^{((4mn-3m-2n+1)t-7mn+5m+4n-2)\alpha}$$

$$MZ_2^\alpha(G) = \left(\frac{2^{20\alpha}}{3^{12\alpha}}\right) \left(\frac{2^{4\alpha}}{3^{6\alpha}}\right)^m \left(\frac{3^{6\alpha}}{2^{16\alpha}}\right)^{mn} \left(\frac{2^{16mn+4n-16}}{3^{-6(m+1)}}\right)^{t\alpha}. \quad \square$$

Corollary 2.1. *If we put $\alpha = 2$, the close result of multiplicative general first and second Zagreb index gives multiplicative first and second hyper Zagreb index.*

$$MZ_1^2(G) = HII_1(G) = \left(\frac{7^{24}}{(2^{16})(3^{20})(5^8)}\right) \left(\frac{(2^{16})(5^8)}{(3^8)(7^8)}\right)^n \left(\frac{(2^{46})(3^{16})}{7^{28}}\right)^m \left(\frac{7^{12}}{2^{42}}\right)^{mn} \left((2^{24mn-18m-4n+6})(3^{8n})(5^8)(7^{12(m-1)})\right)^t$$

$$MZ_2^2(G) = HII_2(G) = \left(\frac{2^{40}}{3^{24}}\right) \left(\frac{2^8}{3^{12}}\right)^m \left(\frac{3^{12}}{2^{32}}\right)^{mn} \left(\frac{2^{32mn+8n-32}}{3^{-12(m+1)}}\right)^t.$$

The multiplicative sum connectivity index and multiplicative product connectivity index of copper graphite $CG(m, n)$ is computed in the following theorem.

Theorem 2.3. *Consider the graph of $G \cong CG(n)$ with $m, n > 1$ and $t \geq 3$, then its multiplicative sum and product connectivity index is equal to*

$$XII(G) = \left(\frac{(2^4)(3^5)(5^2)}{7^6}\right) \left(\frac{(3^2)(7^2)}{(2^4)(5^2)}\right)^n \left(\frac{7^7}{(2^{\frac{7}{2}})(3^4)}\right)^m \left(\frac{1}{(2^{6mn-n-\frac{9m}{2}+\frac{3}{2}})(3^{2n})(5^2)(7^{3(m-1)})}\right)^t \left(\frac{2^{\frac{21}{2}}}{7^3}\right)^{mn}$$

$$\chi II(G) = \left(\frac{3^6}{2^{10}}\right) \left(\frac{3^3}{2^2}\right)^m \left(\frac{1}{(2^{8mn+2n-8})(3^{3(m-1)})}\right)^t \left(\frac{2^8}{3^3}\right)^{mn}.$$

Proof. Let chemical graph $G \cong CG(m, n)$ with $m, n > 1$ and $t \geq 3$. The above result can be proved by using Table 1 in the following computation. The multiplicative sum connectivity index is computed as below:

$$XII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}$$

$$XII(G) = \left(\frac{1}{\sqrt{2+2}}\right)^4 \times \left(\frac{1}{\sqrt{2+3}}\right)^{4(n+t-1)} \times \left(\frac{1}{\sqrt{2+4}}\right)^{4(m-n-t+nt)} \times \left(\frac{1}{\sqrt{3+3}}\right)^{4m+4t-10} \times \left(\frac{1}{\sqrt{3+4}}\right)^{6mn+6mt-14m-4n-6t+12} \times \left(\frac{1}{\sqrt{4+4}}\right)^{(4mn-3m-2n+1)t-7mn+5m+4n-2}$$

$$XII(G) = \left(\frac{(2^4)(3^5)(5^2)}{7^6}\right) \left(\frac{(3^2)(7^2)}{(2^4)(5^2)}\right)^n \left(\frac{7^7}{(2^{\frac{7}{2}})(3^4)}\right)^m \left(\frac{1}{(2^{6mn-n-\frac{9m}{2}+\frac{3}{2}})(3^{2n})(5^2)(7^{3(m-1)})}\right)^t \left(\frac{2^{\frac{21}{2}}}{7^3}\right)^{mn}.$$

The multiplicative product connectivity index is computed as below:

$$\chi II(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) \times d_G(v)}}$$

$$\chi II(G) = \left(\frac{1}{\sqrt{2 \times 2}}\right)^4 \times \left(\frac{1}{\sqrt{2 \times 3}}\right)^{4(n+t-1)} \times \left(\frac{1}{\sqrt{2 \times 4}}\right)^{4(m-n-t+nt)} \times \left(\frac{1}{\sqrt{3 \times 3}}\right)^{4m+4t-10} \times$$

$$\left(\frac{1}{\sqrt{3 \times 4}}\right)^{6mn+6mt-14m-4n-6t+12} \times \left(\frac{1}{\sqrt{4 \times 4}}\right)^{(4mn-3m-2n+1)t-7mn+5m+4n-2}$$

$$\chi II(G) = \left(\frac{3^6}{2^{10}}\right) \left(\frac{3^3}{2^2}\right)^m \left(\frac{1}{(2^{8mn+2n-8})(3^{3(m-1)})}\right)^t \left(\frac{2^8}{3^3}\right)^{mn}. \quad \square$$

Now, we compute multiplicative atom bond connectivity index of copper graphite $CG(m, n)$ in the following theorem.

Theorem 2.4. Consider the graph of $G \cong CG(m, n)$ with $m, n > 1$ and $t \geq 3$, then its multiplicative atom bond connectivity index is equal to

$$ABCH(G) = \left(\frac{(3^3)(5^6)}{2^{19}}\right) \left(\frac{(3^4)}{(2^2)(5^2)}\right)^n \left(\frac{(2^{\frac{17}{2}})(3^{\frac{11}{2}})}{5^7}\right)^m \left(\frac{(2^{n-\frac{3m}{2}-6mn+\frac{17}{2}})(5^{3(m-1)})}{3^{n+\frac{9m}{2}-2mn+\frac{1}{2}}}\right)^t$$

$$\left(\frac{(2^{\frac{9}{2}})(5^3)}{3^{\frac{13}{2}}}\right)^{mn}.$$

Proof. Let chemical graph $G \cong CG(m, n)$ with $m, n > 1$ and $t \geq 3$. The above result can be proved by using Table 1 in the following computation. The multiplicative atom bond connectivity index is computed as below:

$$ABCH(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u) \times d_G(v)}}$$

$$ABCH(G) = \left(\sqrt{\frac{2+2-2}{2 \times 2}}\right)^4 \times \left(\sqrt{\frac{2+3-2}{2 \times 3}}\right)^{4(n+t-1)} \times \left(\sqrt{\frac{2+4-2}{2 \times 4}}\right)^{4(m-n-t+nt)}$$

$$\times \left(\sqrt{\frac{3+3-2}{3 \times 3}}\right)^{4m+4t-10} \times \left(\sqrt{\frac{3+4-2}{3 \times 4}}\right)^{6mn+6mt-14m-4n-6t+12}$$

$$\times \left(\sqrt{\frac{4+4-2}{4 \times 4}}\right)^{(4mn-3m-2n+1)t-7mn+5m+4n-2}.$$

After some calculation and simplification, we have

$$ABCH(G) = \left(\frac{(3^3)(5^6)}{2^{19}}\right) \left(\frac{(3^4)}{(2^2)(5^2)}\right)^n \left(\frac{(2^{\frac{17}{2}})(3^{\frac{11}{2}})}{5^7}\right)^m \left(\frac{(2^{n-\frac{3m}{2}-6mn+\frac{17}{2}})(5^{3(m-1)})}{3^{n+\frac{9m}{2}-2mn+\frac{1}{2}}}\right)^t$$

$$\left(\frac{(2^{\frac{9}{2}})(5^3)}{3^{\frac{13}{2}}}\right)^{mn}. \quad \square$$

The multiplicative general geometric arithmetic index and multiplicative geometric arithmetic index of copper graphite $CG(m, n)$ is discussed in the following theorem.

Theorem 2.5. Consider the graph of $G \cong CG(m, n)$ with $n > 1$ and $t \geq 3$, then its general multiplicative geometric arithmetic and multiplicative geometric arithmetic index is equal to

$$GA^\alpha II(G) = \left(\frac{(2^{18})(3^4)(5^4)}{7^{12}} \right)^\alpha \left(\frac{(3^4)(7^4)}{(2^8)(5^4)} \right)^{n\alpha} \left(\frac{7^{14}}{(2^{22})(3^{11})} \right)^{m\alpha} \left(\frac{(2^{12mn+6n-12})(3^{3m-4n+3})}{(5^4)(7^{6(m-1)})} \right)^{t\alpha} \left(\frac{(2^{12})(3^3)}{7^6} \right)^{mn\alpha}$$

$$GAII(G) = \left(\frac{(2^{18})(3^4)(5^4)}{7^{12}} \right) \left(\frac{(3^4)(7^4)}{(2^8)(5^4)} \right)^n \left(\frac{7^{14}}{(2^{22})(3^{11})} \right)^m \left(\frac{(2^{12mn+6n-12})(3^{3m-4n+3})}{(5^4)(7^{6(m-1)})} \right)^t \left(\frac{(2^{12})(3^3)}{7^6} \right)^{mn}.$$

Proof. Let chemical graph $G \cong CG(m, n)$ with $m, n > 1$ and $t \geq 3$. The above result can be proved by using Table 1 in the following computation. The multiplicative general geometric arithmetic index is computed as below:

$$GA^\alpha II(G) = \prod_{uv \in E(G)} \left(\frac{2\sqrt{d_G(u) \times d_G(v)}}{d_G(u) + d_G(v)} \right)^\alpha$$

$$GA^\alpha II(G) = \left(\frac{2\sqrt{2 \times 2}}{2+2} \right)^{4\alpha} \times \left(\frac{2\sqrt{2 \times 3}}{2+3} \right)^{4n\alpha} \times \left(\frac{2\sqrt{2 \times 3}}{2+3} \right)^{4(t-1)\alpha} \times \left(\frac{2\sqrt{2 \times 4}}{2+4} \right)^{4(m-n-t+nt)\alpha} \times$$

$$\left(\frac{2\sqrt{3 \times 3}}{3+3} \right)^{(4m+4t-10)\alpha} \times \left(\frac{2\sqrt{3 \times 4}}{3+4} \right)^{(6mn+6mt-14m-4n-6t+12)\alpha}$$

$$\times \left(\frac{2\sqrt{4 \times 4}}{4+4} \right)^{((4mn-3m-2n+1)t-7mn+5m+4n-2)\alpha}.$$

After some calculation and simplification, we have

$$GA^\alpha II(G) = \left(\frac{(2^{18})(3^4)(5^4)}{7^{12}} \right)^\alpha \left(\frac{(3^4)(7^4)}{(2^8)(5^4)} \right)^{n\alpha} \left(\frac{7^{14}}{(2^{22})(3^{11})} \right)^{m\alpha} \left(\frac{(2^{12mn+6n-12})(3^{3m-4n+3})}{(5^4)(7^{6(m-1)})} \right)^{t\alpha} \left(\frac{(2^{12})(3^3)}{7^6} \right)^{mn\alpha}.$$

The multiplicative geometric arithmetic index can be computed by putting $\alpha = 1$ in above equation.

$$GAII(G) = \left(\frac{(2^{18})(3^4)(5^4)}{7^{12}} \right) \left(\frac{(3^4)(7^4)}{(2^8)(5^4)} \right)^n \left(\frac{7^{14}}{(2^{22})(3^{11})} \right)^m \left(\frac{(2^{12mn+6n-12})(3^{3m-4n+3})}{(5^4)(7^{6(m-1)})} \right)^t \left(\frac{(2^{12})(3^3)}{7^6} \right)^{mn}.$$

□

3. Conclusion. The purpose of this article is to discuss the multiplicative indices of the chemical graph of carbon graphite. The degree based multiplicative indices for the chemical graph of carbon graphite have been computed of chemical graph of $CG(m, n)$ t -levels. The whole article is distributed into two catagories.

Firstly, the chemical structure of carbon graphite has been studied and discussed in detail with its edge partition technique. Secondly, the multiplicative first and second Zagreb index, multiplicative hyper first and second Zagreb index, general multiplicative first and second Zagreb index, multiplicative product connectivity index, multiplicative sum connectivity index, multiplicative *ABC* index, multiplicative *GA* index and general multiplicative *GA* index have been computed for $CG(m, n)$ t -levels. The multiplicative indices can be used in the study of QSPR/QSAR and it plays a prominent role in analyzing the boiling point and melting point for medicinal drugs, mathematical chemistry and chemical structures. Our multiplicative indices results will illustrate the promising prospects of application for medical, biological and chemical sciences.

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