METASTRUCTURAL IDENTIFICATION AND NEIGHBORHOOD SYSTEMS

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ABSTRACT

In this paper, the problem of structural identification of neighborhood systems is discussed. The concept of metastructural identification as the first stage of structural identification is proposed. The neighborhood metasystems are defined as a result of metastructural identification. Two types of neighborhood metasystems are considered: vertex and relational. Examples of vertex and relational metasystems over a neighborhood structure are given.

Keywords: metastructural, identification, neighborhood

INTRODUCTION

Systems on graphs, or systems of equations associated with graphs, often appear in applications in some version (see, for example, [1] or [2]), but these versions, as a rule, reflect the specifics of the corresponding applications. In [3] and [4], *neighborhood systems* were defined which are a fairly general class of systems on graphs, and a considerable number of publications were devoted to it. We note that in the earlier work [3] the term "neighborhood systems" was absent, but the corresponding classes of systems and graphs were already discussed. The term "neighborhood systems" appeared in [4], but in this work there was no description of the graphs associated with such systems, although it was intended. Later, beginning with [5], attention was shifted to these graphs, which were called *neighborhood structures*, and different types of neighborhood systems were already considered as superstructures over neighborhood structures. The definition of the neighborhood structure was subsequently modified in [6] and [7], the final version was proposed in [8]. In parallel, two classes of systems over neighborhood structures were defined: *vertex systems*, when the equations of the system correspond to the vertices of the structure, and *relational systems*, when the equations correspond to the arcs; in both cases, the systems can be either static or dynamic. In this case, the neighborhood systems from [3] and [4] were interpreted as static vertex systems.

NEIGHBORHOOD SYSTEMS IN THE STATE SPACE

In [3] and [4] the problem of generalization of classical discrete control systems

$$\begin{cases} X^{t+1} = F(X^t, U^t) \\ W^{t+1} = C(X^t, U^t) \end{cases} \quad (1)$$

to the case of distributed systems was considered. The system (1) was assumed to be given on the finite subgraph $G_{[0,T]}$ of infinite graph $G_{\mathbb{Z}}$ over $\mathbb{Z}$ with arcs $(n, n+1)$. The idea of generalization consisted in replacing the digraph $G_{[0,T]}$ with an arbitrary digraph. The discrete-time moments $t \in \{0, \dots, T\}$ were replaced by elements v of a finite set V (nodes), the state vectors X^t and the inputs vectors U^t at time t - by the state vectors X^v and inputs vectors U^v

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at the nodes v , and the system of the equation (1) - by a multisystem, consisting of $|V|$ subsystems, corresponding to the nodes $v \in V$ and including the variables of states and inputs (controls) from the neighborhoods by states and controls of these nodes. These neighborhoods were given by two digraphs G_X and G_U over V which, as already mentioned, were explicitly described in [3] and were implicitly present in [4]. In fact, such a system, written in a coordinate form or, in other words, "in the state space", can be interpreted as a system of equations

$$\begin{cases} \tilde{X} = \tilde{F}(\tilde{X}, \tilde{U}) \\ \tilde{W} = \tilde{C}(\tilde{X}, \tilde{U}) \end{cases} \quad \text{or, in implicit form} \quad \begin{cases} \hat{F}(\tilde{X}, \tilde{U}) = 0 \\ \hat{C}(\tilde{X}, \tilde{U}) = 0 \end{cases}$$

for stationary states of a discrete dynamic model

$$\begin{cases} \tilde{X}^{t+1} = \tilde{F}(\tilde{X}^t, \tilde{U}^t) = \tilde{X}^t + \hat{F}(\tilde{X}^t, \tilde{U}^t) \\ \tilde{W}^{t+1} = \tilde{C}(\tilde{X}^t, \tilde{U}^t) = \tilde{W}^t + \hat{C}(\tilde{X}^t, \tilde{U}^t) \end{cases}$$

of the type (1), where "multi-vector" variables $\tilde{X}$ and $\tilde{U}$ contain all the state vectors X^v and all input vectors U^v , while the digraphs G_X and G_U structurize the system, that is, determine for each of the equations the set of variables entering into this equation.

NEIGHBORHOOD SYSTEMS AND NEIGHBORHOOD STRUCTURES

The interpretation of neighborhood systems described above allows us to consider the associated structural graphs not as an alternative to the time variable, but as a means of describing the occurrence of spatial variables in the equations of the system (1). Thus, the main object of study become the neighborhood structures (digraphs) and the vertex and relational systems associated with them. These systems, in contrast to the static neighborhood systems of [4], already have a dynamic origin, since the time variable T is conserved, not dissolved among the vertices of the graph V . Static systems in this case arise, as usual, in the form of models for stationary states of dynamical systems. Of course, the dynamics over time can also be introduced in the class of neighborhood systems from [4] but, in order to remain formally in this class, it is necessary to go from the graphs G_X and G_U to the countable graphs $G_{X \times \mathbb{Z}}$ and $G_{U \times \mathbb{Z}}$. In the new interpretation of neighborhood systems, such a transition also makes sense, but acts in the opposite direction, transforming dynamic (vertex or relational) systems into static ones, and can be regarded as an analogue of the transition to the extended phase space. We can point out another difference between the new ideology and the old one: instead of two digraphs G_X and G_U over V , we consider one graph over $\hat{V} = U \sqcup V \sqcup W$, where U and W are inputs and outputs, which corresponds to the structure of the system (1).

Fig. 1. Neighborhood structure

An example of a neighborhood structure is shown in Fig. 1. In this figure, vertices 1 and 2 are inputs, vertices 8,9,10 - outputs, vertices 3,4,5,6,7 - internal nodes, nodes 4,5 and 6 have loops (self-actions). Node 5 is, in our terminology, a *reflexive output*.

METASTRUCTURAL IDENTIFICATION

In connection with the above changes in the viewpoint on neighborhood systems, we actually return to the problem of modeling objects by classical systems of the form (1), but in this case, as the first stage of modeling, we consider the task of metastructural identification, having in mind the definition of model nodes, links between nodes and sets of corresponding variables. This task, of course, is always solved in one way or another in the modeling process, and in this sense there is nothing fundamentally new here. However, we consider it useful to separate and to a certain extent formalize this "metastructural" stage of the modeling of the object. Of course, any formalization of this stage will to some extent limit the class of models, and our "vertex-relational" is not an exception, but this is an inevitable property of any formalization. Let us explain the origin of the term "metastructural identification". Structural identification of the modeled system, as a rule, can be divided into two stages. At the first stage we define the nodes of the model, the connections between them and the sets of variables corresponding to these nodes and connections. This information can be represented in an equivalent way as a *metasystem*, which is a prototype for the final analytical system. The prototype (metasystem) defines sets of variables and their occurrences in the equations of the model. In the second stage we choose the analytical type of equations model and, if possible, minimize the number of unknown parameters that are subject to further parametric identification. We retain the term "structural identification" for the second stage, while the former is called "metastructural identification" and we define it as the construction of the neighborhood structure (digraph), the specification of model variables, which we call the *equipment* of the structure, and the definition of the type of interactions between the nodes of the structure. Our experience in mathematical modeling shows that in many cases it makes sense to distinguish between two types of such interactions. For the *vertex-type* the equations of the model specify the states of the nodes of the structure depending on the states of the inputs and all outputs from the nodes coincide with the states of the nodes. For the *relational-type*, the equations of the model specify the states of the outputs from the nodes (different for different outputs), depending on the states of the inputs to these nodes. In the first case, the equations and equipment correspond to the nodes of the neighborhood structure, in the second case – to its arcs. Metastructural identification is considered complete in those cases when all the variables of the model (equipment) become metascalar, that is, they are used in the equations as single symbols, regardless of their arbitrarily complex internal structure (vector, matrix, etc.). For the vertex and relational systems over a given

neighborhood structure, the concept of a metagraphs turned out to be very convenient (see [9]). In [8] we defined these two types of models using metagraphs and described the connections between these models. We note that the ideology of metastructural identification remains valid also for continuous systems.

EXAMPLES

The dynamic vertex metasystem corresponding to the neighborhood structure in Fig. 1 has the form:

$$\begin{cases} X^{t+1}(3) = F_3(U^t(1), X^t(4)) \\ X^{t+1}(4) = F_4(U^t(1), U^t(2), X^t(3), X^t(4), X^t(7)) \\ X^{t+1}(5) = F_5(U^t(2), X^t(5)) \\ X^{t+1}(6) = F_6(X^t(4), X^t(6)) \\ X^{t+1}(7) = F_7(X^t(6)) \\ W^{t+1}(8) = C_8(X(6)) \\ W^{t+1}(9) = C_9(X(6)) \\ W^{t+1}(10) = C_{10}(U(2), X(7)) \end{cases}$$

The dynamic relational metasystem corresponding to the neighborhood structure in Fig. 1 has the form:

$$\begin{aligned} Y^{t+1}(3,4) &= F_{3,4}(U^t(1,3), Y^t(4,3)) \\ \begin{cases} Y^{t+1}(4,3) = F_{4,3}(U^t(1,4), U^t(2,4), Y^t(3,4), Y^t(4,4), Y^t(7,4)) \\ Y^{t+1}(4,4) = F_{4,4}(U^t(1,4), U^t(2,4), Y^t(3,4), Y^t(4,4), Y^t(7,4)) \\ Y^{t+1}(4,6) = F_{4,6}(U^t(1,4), U^t(2,4), Y^t(3,4), Y^t(4,4), Y^t(7,4)) \end{cases} \\ Y^{t+1}(5,5) &= F_{5,5}(U^t(2,5), Y^t(5,5)) \\ \begin{cases} Y^{t+1}(6,6) = F_{6,6}(Y^t(4,6), Y^t(6,6)) \\ Y^{t+1}(6,7) = F_{6,7}(Y^t(4,6), Y^t(6,6)) \\ Y^{t+1}(6,8) = F_{6,8}(Y^t(4,6), Y^t(6,6)) \\ Y^{t+1}(6,9) = F_{6,9}(Y^t(4,6), Y^t(6,6)) \end{cases} \\ \begin{cases} Y^{t+1}(7,4) = F_{7,4}(Y^t(6,7)) \\ Y^{t+1}(7,10) = F_{7,10}(Y^t(6,7)) \end{cases} \\ W^{t+1}(8) &= C_8(Y(6,8)) \\ W^{t+1}(9) &= C_9(Y(6,9)) \\ W^{t+1}(10) &= C_{10}(U(1,10), Y(7,1)) \end{aligned}$$

CONCLUSION

The proposed concept of metastructural identification includes the construction of a neighborhood structure (digraph), the definition of equipping variables and the type of interactions between the nodes of the structure. Such an identification can be considered as the first stage of structural identification. The result of metastructural identification is a metasytem (static or dynamic, discrete or continuous). This metasytem is a useful prototype for construction a further analytical model.

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