

Discussiones Mathematicae

Probability and Statistics 29 (2009) 131–153

doi:10.7151/dmps.1111

SOME APPLICATIONS OF PROBABILITY GENERATING FUNCTION BASED METHODS TO STATISTICAL ESTIMATION

MANUEL L. ESQUÍVEL

*Departamento de Matemática
Faculdade de Ciências e Tecnologia
Universidade Nova de Lisboa, Portugal*

e-mail: mle@fct.unl.pt

This work is dedicated to my dearest friend and colleague João Tiago Mexia, as a token of everlasting admiration, respect and gratitude, being certain that his unbreakable enthusiasm for Mathematics and mathematicians will always be, for us, a source of inspiration and guidance.

Abstract

After recalling previous work on probability generating functions for real valued random variables we extend to these random variables uniform laws of large numbers and functional limit theorem for the empirical probability generating function. We present an application to the study of continuous laws, namely, estimation of parameters of Gaussian, gamma and uniform laws by means of a minimum contrast estimator that uses the empirical probability generating function of the sample. We test the procedure by simulation and we prove the consistency of the estimator.

Keywords: probability generating function, empirical laws, estimation of parameters of continuous laws.

2000 Mathematics Subject Classification: 60–08, 60E10, 30B50.

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Received 11 October 2009