ALTERNATING-PANCYCLISM IN 2-EDGE-COLORED GRAPHS¹

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Abstract

An *alternating cycle* in a 2-edge-colored graph is a cycle such that any two consecutive edges have different colors. Let $G_1, \dots, G_k$ be a collection of pairwise vertex disjoint 2-edge-colored graphs. The *colored generalized sum* of $G_1, \dots, G_k$, denoted by $\oplus_{i=1}^k G_i$, is the set of all 2-edge-colored graphs G such that: (i) $V(G) = \bigcup_{i=1}^k V(G_i)$, (ii) $G[V(G_i)] \cong G_i$ for $i = 1, \dots, k$ where $G[V(G_i)]$ has the same coloring as G_i and (iii) between each pair of vertices in different summands of G there is exactly one edge, with an arbitrary but fixed color. A graph G in $\oplus_{i=1}^k G_i$ will be called a *colored generalized sum* (c.g.s.) and we will say that $e \in E(G)$ is an *exterior edge* if and only if $e \in E(G) \setminus \left(\bigcup_{i=1}^k E(G_i)\right)$. The set of exterior edges will be denoted by $E_{\oplus}$. A 2-edge-colored graph G of order $2n$ is said to be an *alternating-pancyclic graph*, whenever for each $l \in \{2, \dots, n\}$, there exists an alternating cycle of length $2l$ in G .

The topics of pancyclism and vertex-pancyclicity are deeply and widely studied by several authors. The existence of alternating cycles in 2-edge-colored graphs has been studied because of its many applications. In this paper, we give sufficient conditions for a graph $G \in \oplus_{i=1}^k G_i$ to be an alternating-pancyclic graph.

Keywords: 2-edge-colored graph, alternating cycle, alternating-pancyclic graph.

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