PROPER RAINBOW CONNECTION NUMBER OF GRAPHS

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Abstract

A path in an edge-coloured graph is called a *rainbow path* if its edges receive pairwise distinct colours. An edge-coloured graph is said to be *rainbow connected* if any two distinct vertices of the graph are connected by a rainbow path. The minimum k for which there exists such an edge-colouring is the rainbow connection number $rc(G)$ of G . Recently, Bau *et al.* [*Rainbow connectivity in some Cayley graphs*, Australas. J. Combin. 71 (2018) 381–393] introduced this concept with the additional requirement that the edge-colouring must be proper. The *proper rainbow connection number* of G , denoted by $prc(G)$, is the minimum number of colours needed in order to make it properly rainbow connected. Obviously, $prc(G) \geq \max\{rc(G), \chi'(G)\}$.

In this paper we first prove an improved upper bound $prc(G) \leq n$ for every connected graph G of order $n \geq 3$. Next we show that the difference $prc(G) - \max\{rc(G), \chi'(G)\}$ can be arbitrarily large. Finally, we present several sufficient conditions for graph classes satisfying $prc(G) = \chi'(G)$.

Keywords: edge-colouring, rainbow connection number, proper rainbow connection number.

2010 Mathematics Subject Classification: 05C15, 05C40, 05C07.

¹Part of this research was done while the first author was a postdoctoral fellow at the Institute of Mathematics, Vietnam Academy of Science and Technology and he wishes to express gratitude for the very good conditions and the pleasant environments of work there. The first author is also thankful to Prof. Ngo Dac Tan, Prof. Phan Thi Ha Duong, and other people for their help at the time. This research is supported by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 08/2020/STS01.

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Received 2 December 2019

Revised 22 April 2020

Accepted 22 April 2020