ON ANTIPODAL AND DIAMETRICAL PARTIAL CUBES

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Abstract

We prove that any diametrical partial cube of diameter at most 6 is antipodal. Because any antipodal graph is harmonic, this gives a partial answer to a question of Fukuda and Handa [*Antipodal graphs and oriented matroids*, Discrete Math. 111 (1993) 245–256] whether any diametrical partial cube is harmonic, and improves a previous result of Klavžar and Kovše [*On even and harmonic-even partial cubes*, Ars Combin. 93 (2009) 77–86].

Keywords: diametrical graph, harmonic graph, antipodal graph, partial cube, diameter, isometric dimension.

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