THE GENERAL POSITION PROBLEM ON KNESER GRAPHS AND ON SOME GRAPH OPERATIONS

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Abstract

A vertex subset S of a graph G is a general position set of G if no vertex of S lies on a geodesic between two other vertices of S . The cardinality of a largest general position set of G is the general position number (gp-number) $\text{gp}(G)$ of G . The gp-number is determined for some families of Kneser graphs, in particular for $K(n, 2)$, $n \geq 4$, and $K(n, 3)$, $n \geq 9$. A sharp lower bound on the gp-number is proved for Cartesian products of graphs. The gp-number is also determined for joins of graphs, coronas over graphs, and line graphs of complete graphs.

Keywords: general position set, Kneser graphs, Cartesian product of graphs, corona over graphs, line graphs.

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