TOTAL DOMINATION IN GENERALIZED PRISMS AND A NEW DOMINATION INVARIANT

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Abstract

In this paper we complement recent studies on the total domination of prisms by considering generalized prisms, i.e., Cartesian products of an arbitrary graph and a complete graph. By introducing a new domination invariant on a graph G , called the k -rainbow total domination number and denoted by $\gamma_{\text{krt}}(G)$, it is shown that the problem of finding the total domination number of a generalized prism $G \square K_k$ is equivalent to an optimization problem of assigning subsets of $\{1, 2, \dots, k\}$ to vertices of G . Various properties of the new domination invariant are presented, including, inter alia, that $\gamma_{\text{krt}}(G) = n$ for a nontrivial graph G of order n as soon as $k \geq 2\Delta(G)$. To prove the mentioned result as well as the closed formulas for the k -rainbow total domination number of paths and cycles for every k , a new weight-redistribution method is introduced, which serves as an efficient tool for establishing a lower bound for a domination invariant.

Keywords: domination, k -rainbow total domination, total domination.

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REFERENCES

- [1] H.A. Ahangar, J. Amjadi, N. Jafari Rad and V. Samodivkin, *Total k -Rainbow domination numbers in graphs*, Commun. Comb. Optim. **3** (2018) 37–50.
<https://doi.org/10.22049/CCO.2018.25719.1021>
- [2] J. Amjadi, N. Dehgardi, M. Furuya and S.M. Sheikholeslami, *A sufficient condition for large rainbow domination number*, Int. J. Comput. Math. Comput. Sys. Theory **2** (2017) 53–65.
<https://doi.org/10.1080/23799927.2017.1330282>

- [3] J. Azarija, M.A. Henning and S. Klavžar, (*Total*) *domination in prisms*, Electron. J. Combin. **24** (2017) #P1.19.
<https://doi.org/10.327236/6288>
- [4] B. Brešar, T.R. Hartinger, T. Kos and M. Milanič, *On total domination in the Cartesian product of graphs*, Discuss. Math. Graph Theory **38** (2018) 963–976.
<https://doi.org/10.7151/dmgt.2039>
- [5] B. Brešar, *Vizing’s conjecture for graphs with domination number 3—a new proof*, Electron. J. Combin. **22** (2015) #P3.38.
<https://doi.org/10.37236/5182>
- [6] B. Brešar, *Improving the Clark-Suen bound on the domination number of the Cartesian product of graphs*, Discrete Math. **340** (2017) 2398–2401.
<https://doi.org/10.1016/j.disc.2017.05.007>
- [7] B. Brešar, P. Dorbec, W. Goddard, B.L. Hartnell, M.A. Henning, S. Klavžar and D.F. Rall, *Vizing’s conjecture: A survey and recent results*, J. Graph Theory **69** (2012) 46–76.
<https://doi.org/10.1002/jgt.20565>
- [8] B. Brešar, M.A. Henning and D.F. Rall, *Rainbow domination in graphs*, Taiwanese J. Math. **12** (2008) 213–225.
<https://doi.org/10.11650/twjtm/1500602498>
- [9] B. Brešar and T.K. Šumenjak, *On the 2-rainbow domination in graphs*, Discrete Appl. Math. **155** (2007) 2394–2400.
<https://doi.org/10.1016/j.dam.2007.07.018>
- [10] S. Brezovnik and T.K. Šumenjak, *Complexity of k -rainbow independent domination and some results on the lexicographic product of graphs*, Appl. Math. Comput. **349** (2019) 214–220.
<https://doi.org/10.1016/j.amc.2018.12.009>
- [11] K. Choudhary, S. Margulies and I.V. Hicks, *A note on total and paired domination of Cartesian product graphs*, Electron. J. Combin. **20** (2013) #P25.
<https://doi.org/10.37236/2535>
- [12] W. Desormeaux and M.A. Henning, *Paired domination in graphs: A survey and recent results*, Util. Math. **94** (2014) 101–166.
- [13] W. Goddard and M.A. Henning, *A note on domination and total domination in prisms*, J. Comb. Optim. **35** (2018) 14–20.
<https://doi.org/10.1007/s10878-017-0150-0>
- [14] M.A. Henning and D.F. Rall, *On the total domination number of Cartesian products of graphs*, Graphs Combin. **21** (2005) 63–69.
<https://doi.org/10.1007/s00373-004-0586-8>
- [15] M.A. Henning and A. Yeo, *Total Domination in Graphs* (Springer, New York, 2013).
<https://doi.org/10.1007/978-1-4614-6525-6>
- [16] P.T. Ho, *A note on the total domination number*, Util. Math. **77** (2008) 97–100.

- [17] P. Kumbargoudra and J.V. Kureethara, *Total k -rainbow domination in graphs*, Int. J. Civ. Eng. Technol. **8** (2017) 867–875.
- [18] Y. Lu and X. Hou, *Total domination in the Cartesian product of a graph and K_2 or C_n* , Util. Math. **83** (2010) 313–322.
- [19] Z. Shao, S.M. Sheikholeslamib, B. Wang, P. Wu and X. Zhang, *Trees with equal total domination and 2-rainbow domination numbers*, Filomat **32** (2018) 599–607.
<https://doi.org/10.2298/FIL1802599S>
- [20] T.K. Šumenjak, D.F. Rall and A. Tepeh, *On k -rainbow independent domination in graphs*, Appl. Math. Comput. **333** (2018) 353–361.
<https://doi.org/10.1016/j.amc.2018.03.113>

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