A NOTE ON THE EQUITABLE CHOOSABILITY OF COMPLETE BIPARTITE GRAPHS

JEFFREY A. MUDROCK, MADELYNN CHASE, EZEKIEL THORNBURGH

ISAAC KADERA AND TIM WAGSTROM

*Department of Mathematics
College of Lake County
Grayslake, IL 60030*

e-mail: jumudrock@clcillinois.edu

Abstract

In 2003 Kostochka, Pelsmayer, and West introduced a list analogue of equitable coloring called equitable choosability. A k -assignment, L , for a graph G assigns a list, $L(v)$, of k available colors to each $v \in V(G)$, and an equitable L -coloring of G is a proper coloring, f , of G such that $f(v) \in L(v)$ for each $v \in V(G)$ and each color class of f has size at most $\lceil |V(G)|/k \rceil$. Graph G is said to be equitably k -choosable if an equitable L -coloring of G exists whenever L is a k -assignment for G . In this note we study the equitable choosability of complete bipartite graphs. A result of Kostochka, Pelsmayer, and West implies $K_{n,m}$ is equitably k -choosable if $k \geq \max\{n, m\}$ provided $K_{n,m} \neq K_{2l+1, 2l+1}$. We prove $K_{n,m}$ is equitably k -choosable if $m \leq \lceil (m+n)/k \rceil (k-n)$ which gives $K_{n,m}$ is equitably k -choosable for certain k satisfying $k < \max\{n, m\}$. We also give a complete characterization of the equitable choosability of complete bipartite graphs that have a partite set of size at most 2.

Keywords: graph coloring, equitable coloring, list coloring, equitable choosability.

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