NEIGHBOR PRODUCT DISTINGUISHING TOTAL
COLORINGS OF PLANAR GRAPHS WITH
MAXIMUM DEGREE AT LEAST TEN¹

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Abstract

A proper $[k]$ -total coloring c of a graph G is a proper total coloring c of G using colors of the set $[k] = \{1, 2, \dots, k\}$. Let $p(u)$ denote the product of the color on a vertex u and colors on all the edges incident with u . For each edge $uv \in E(G)$, if $p(u) \neq p(v)$, then we say the coloring c distinguishes adjacent vertices by product and call it a neighbor product distinguishing k -total coloring of G . By $\chi''_{\Pi}(G)$, we denote the smallest value of k in such a coloring of G . It has been conjectured by Li *et al.* that $\Delta(G) + 3$ colors enable the existence of a neighbor product distinguishing total coloring. In this paper, by applying the Combinatorial Nullstellensatz, we obtain that the conjecture holds for planar graph with $\Delta(G) \geq 10$. Moreover, for planar graph G with $\Delta(G) \geq 11$, it is neighbor product distinguishing $(\Delta(G) + 2)$ -total colorable, and the upper bound $\Delta(G) + 2$ is tight.

Keywords: total coloring, neighbor product distinguishing coloring, planar graph.

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