THE SPECTRUM PROBLEM FOR THE CONNECTED CUBIC GRAPHS OF ORDER 10

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Abstract

We show that if G is a connected cubic graph of order 10, then there exists a G -decomposition of K_v if and only if $v \equiv 1$ or $10 \pmod{15}$ except when $v = 10$ and G is one of 5 specific graphs.

Keywords: spectrum problem, graph decomposition, cubic graphs.

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