OUTPATHS OF ARCS IN REGULAR 3-PARTITE TOURNAMENTS

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Abstract

Guo [*Outpaths in semicomplete multipartite digraphs*, Discrete Appl. Math. 95 (1999) 273–277] proposed the concept of the outpath in digraphs. An outpath of a vertex x (an arc xy , respectively) in a digraph is a directed path starting at x (an arc xy , respectively) such that x does not dominate the end vertex of this directed path. A k -outpath is an outpath of length k . The outpath is a generalization of the directed cycle. A c -partite tournament is an orientation of a complete c -partite graph.

In this paper, we investigate outpaths of arcs in regular 3-partite tournaments. We prove that every arc of an r -regular 3-partite tournament has 2- (when $r \geq 1$), 3- (when $r \geq 2$), and 5-, 6-outpaths (when $r \geq 3$). We also give the structure of an r -regular 3-partite tournament D with $r \geq 2$ that contains arcs which have no 4-outpaths. Based on these results, we conjecture that for all $k \in \{1, 2, \dots, r-1\}$, every arc of r -regular 3-partite tournaments with $r \geq 2$ has $(3k-1)$ - and $3k$ -outpaths, and it has a $(3k+1)$ -outpath except an r -regular 3-partite tournament.

Keywords: multipartite tournament, regular 3-partite tournament, outpaths.

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