ON SEMISYMMETRIC CUBIC GRAPHS OF ORDER $20p^2$, p PRIME

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Abstract

A simple graph is called semisymmetric if it is regular and edge-transitive but not vertex-transitive. Let p be an arbitrary prime. Folkman proved [*Regular line-symmetric graphs*, J. Combin. Theory 3 (1967) 215–232] that there is no semisymmetric graph of order $2p$ or $2p^2$. In this paper an extension of his result in the case of cubic graphs of order $20p^2$ is given. We prove that there is no connected cubic semisymmetric graph of order $20p^2$ or, equivalently, that every connected cubic edge-transitive graph of order $20p^2$ is necessarily symmetric.

Keywords: edge-transitive graph, vertex-transitive graph, semisymmetric graph, order of a graph, classification of cubic semisymmetric graphs.

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