ON $(p, 1)$ -TOTAL LABELLING OF SOME 1-PLANAR GRAPHS¹

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Abstract

A graph is 1-planar if it can be drawn on the plane so that each edge is crossed by at most one other edge. In this paper, it is proved that the $(p, 1)$ -total labelling number ($p \geq 2$) of every 1-planar graph G is at most $\Delta(G) + 2p - 2$ provided that $\Delta(G) \geq 6p + 7$ or $\Delta(G) \geq 4p + 6$ and G is triangle-free.

Keywords: 1-planar graph, total coloring, $(p, 1)$ -total labelling, structural theorem.

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REFERENCES

- [1] F. Bazzaro, M. Montassier and A. Raspaud, *$(d, 1)$ -total labelling of planar graphs with large girth and high maximum degree*, Discrete Math. **307** (2007) 2141–2151.
doi:10.1016/j.disc.2005.12.059
- [2] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications (North-Holland, New York, 1976).
- [3] O.V. Borodin, *A new proof of the 6 color theorem*, J. Graph Theory **19** (1995) 507–521.
doi:10.1002/jgt.3190190406

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- [4] O.V. Borodin, *Solution of Ringel's problems on the vertex-face coloring of plane graphs and on the coloring of 1-planar graphs*, Diskret. Analiz **41** (1984) 12–26.
- [5] J.R. Griggs and R.K. Yeh, *Labeling graphs with a condition at distance two*, SIAM J. Discrete Math. **5** (1992) 586–595.
doi:10.1137/0405048
- [6] I. Fabrici and T. Madaras, *The structure of 1-planar graphs*, Discrete Math. **307** (2007) 854–865.
doi:10.1016/j.disc.2005.11.056
- [7] F. Havet, *(d, 1)-total labelling of graphs*, in: Workshop on Graphs and Algorithms (Dijon, France, 2003).
- [8] F. Havet and M.-L. Yu, *(d, 1)-total labelling of graphs*, Technical Report, 4650 (INRIA, 2002).
- [9] F. Havet and M.-L. Yu, *(p, 1)-total labelling of graphs*, Discrete Math. **308** (2008) 496–513.
doi:10.1016/j.disc.2007.03.034
- [10] L. Kowalik, J.-S. Sereni and R. Škrekovski, *Total-coloring of plane graphs with maximum degree nine*, SIAM J. Discrete Math. **22** (2008) 1462–1479.
doi:10.1137/070688389
- [11] S.G. Kobourov, G. Liotta and F. Montecchiani, *An annotated bibliography on 1-planarity*, Comput. Sci. Rev. **25** (2017) 49–67.
doi:10.1016/j.cosrev.2017.06.002
- [12] M. Montassier and A. Raspaud, *(d, 1)-total labeling of graphs with a given maximum average degree*, J. Graph Theory **51** (2006) 93–109.
doi:10.1002/jgt.20124
- [13] G. Ringel, *Ein Sechsfarbenproblem auf der Kugel*, Abh. Math. Semin. Univ. Hambg. **29** (1965) 107–117.
doi:10.1007/BF02996313
- [14] L. Sun and J.-L. Wu, *On (p, 1)-total labelling of planar graphs*, J. Comb. Optim. **33** (2017) 317–325.
doi:10.1007/s10878-015-9958-7
- [15] J.-L. Wu and P. Wang, *List-edge and list-total colorings of graphs embedded on hyperbolic surfaces*, Discrete Math. **308** (2008) 6210–6215.
doi:10.1016/j.disc.2007.11.044
- [16] R.K. Yeh, *Labelling Graphs with a Condition at Distance Two*, Ph.D. Thesis (Dept. of Math., Univ. of South Carolina, Columbia, SC, USA, 1990).
- [17] Y. Yu, X. Zhang, G. Wang, G. Liu and J. Li, *(2, 1)-total labelling of planar graphs with large maximum degree*, J. Discrete Math. Sci. Cryptogr. **20** (2017) 1625–1636.
doi:10.1080/09720529.2016.1139850
- [18] X. Zhang, B. Niu and J. Yu, *A structure of 1-planar graph and its applications to coloring problems*, Graphs Combin. **35** (2019) 677–688.
doi:10.1007/s00373-019-02027-0

- [19] X. Zhang and J.-L. Wu, *On edge colorings of 1-planar graphs*, Inform. Process. Lett. **111** (2011) 124–128.
doi:10.1016/j.ipl.2010.11.001
- [20] X. Zhang, J.-L. Wu and G. Liu, *List edge and list total coloring of 1-planar graphs*, Front. Math. China **7** (2012) 1005–1018.
doi:10.1007/s11464-012-0184-7
- [21] X. Zhang, Y. Yu and G. Liu, *On $(p, 1)$ -total labelling of 1-planar graphs*, Cent. Eur. J. Math. **9** (2011) 1424–1434.
doi:10.2478/s11533-011-0092-1

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