***H*-KERNELS IN UNIONS OF *H*-COLORED QUASI-TRANSITIVE DIGRAPHS**

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Abstract

Let H be a digraph (possibly with loops) and D a digraph without loops whose arcs are colored with the vertices of H (D is said to be an H -colored digraph). For an arc (x, y) of D , its color is denoted by $c(x, y)$. A directed path $W = (v_0, \dots, v_n)$ in an H -colored digraph D will be called H -path if and only if $(c(v_0, v_1), \dots, c(v_{n-1}, v_n))$ is a directed walk in H . In W , we will say that there is an obstruction on v_i if $(c(v_{i-1}, v_i), c(v_i, v_{i+1})) \notin A(H)$ (if $v_0 = v_n$ we will take indices modulo n). A subset N of $V(D)$ is said to be an H -kernel in D if for every pair of different vertices in N there is no H -path between them, and for every vertex u in $V(D) \setminus N$ there exists an H -path in D from u to N . Let D be an arc-colored digraph. The color-class digraph of D , $\mathcal{C}_C(D)$, is the digraph such that $V(\mathcal{C}_C(D)) = \{c(a) : a \in A(D)\}$ and $(i, j) \in A(\mathcal{C}_C(D))$ if and only if there exist two arcs, namely (u, v) and (v, w) in D , such that $c(u, v) = i$ and $c(v, w) = j$. The main result establishes that if $D = D_1 \cup D_2$ is an H -colored digraph which is a union of asymmetric quasi-transitive digraphs and $\{V_1, \dots, V_k\}$ is a partition of $V(\mathcal{C}_C(D))$ with a property P^* such that

1. V_i is a quasi-transitive V_i -class for every i in $\{1, \dots, k\}$,
2. either $D[\{a \in A(D) : c(a) \in V_i\}]$ is a subdigraph of D_1 or it is a subdigraph of D_2 for every i in $\{1, \dots, k\}$,
3. D_i has no infinite outward path for every i in $\{1, 2\}$,

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4. every cycle of length three in D has at most two obstructions,
then D has an H -kernel.

Some results with respect to the existence of kernels by monochromatic paths in finite digraphs will be deduced from the main result.

Keywords: quasi-transitive digraph, kernel by monochromatic paths, alternating kernel, H -kernel, obstruction.

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