T -COLORINGS, DIVISIBILITY AND THE CIRCULAR CHROMATIC NUMBER

ROBERT JANCZEWSKI

*Department of Algorithms and Systems Modelling
Gdańsk University of Technology
Narutowicza 11/12, Gdańsk, Poland
e-mail: skalar@eti.pg.gda.pl*

ANNA MARIA TRZASKOWSKA

*Department of Applied Informatics in Management
Gdańsk University of Technology
Narutowicza 11/12, Gdańsk, Poland
e-mail: anna.trzaskowska@pg.edu.pl*

AND

KRZYSZTOF TUROWSKI

*Center for Science of Information
Purdue University West Lafayette, Indiana, USA
e-mail: krzysztof.szymon.turowski@gmail.com*

Abstract

Let T be a T -set, i.e., a finite set of nonnegative integers satisfying $0 \in T$, and G be a graph. In the paper we study relations between the T -edge spans $\text{esp}_T(G)$ and $\text{esp}_{d \odot T}(G)$, where d is a positive integer and

$$d \odot T = \{0 \leq t \leq d(\max T + 1) : d \mid t \Rightarrow t/d \in T\}.$$

We show that $\text{esp}_{d \odot T}(G) = d \text{esp}_T(G) - r$, where r , $0 \leq r \leq d - 1$, is an integer that depends on T and G . Next we focus on the case $T = \{0\}$ and show that

$$\text{esp}_{d \odot \{0\}}(G) = \lceil d(\chi_c(G) - 1) \rceil,$$

where $\chi_c(G)$ is the circular chromatic number of G . This result allows us to formulate several interesting conclusions that include a new formula for the circular chromatic number

$$\chi_c(G) = 1 + \inf \{ \text{esp}_{d \odot \{0\}}(G)/d : d \geq 1 \}$$

and a proof that the formula for the T -edge span of powers of cycles, stated as conjecture in [Y. Zhao, W. He and R. Cao, *The edge span of T -coloring on graph C_n^d* , Appl. Math. Lett. 19 (2006) 647–651], is true.

Keywords: T -coloring, circular chromatic number.

2010 Mathematics Subject Classification: 05C15.

REFERENCES

- [1] M.B. Cozzens and F.S. Roberts, *T-colorings of graphs and the channel assignment problem*, Congr. Numer. **35** (1982) 191–208.
- [2] K. Giaro, R. Janczewski and M. Małafiejski, *A polynomial algorithm for finding T -span of generalized cacti*, Discrete Appl. Math. **129** (2003) 371–382.
doi:10.1016/S0166-218X(02)00575-9
- [3] K. Giaro, R. Janczewski and M. Małafiejski, *The complexity of the T -coloring problem for graphs with small degree*, Discrete Appl. Math. **129** (2003) 361–369.
doi:10.1016/S0166-218X(02)00576-0
- [4] G. Fan, *Circular chromatic number and Mycielski graphs*, Combinatorica **24** (2004) 127–135.
doi:10.1007/s00493-004-0008-9
- [5] W.K. Hale, *Frequency assignment: theory and applications*, Proc. IEEE **68** (1980) 1497–1514.
doi:10.1109/PROC.1980.11899
- [6] R. Janczewski, *Divisibility and T -span of graphs*, Discrete Math. **234** (2001) 171–179.
doi:10.1016/S0012-365X(00)00378-2
- [7] R. Janczewski, *Greedy T -colorings of graphs*, Discrete Math. **309** (2009) 1685–1690.
doi:10.1016/j.disc.2008.01.049
- [8] J.S.-T. Juan, I. Sun and P.-X. Wu, *T -coloring on folded hypercubes*, Taiwanese J. Math. **13** (2009) 1331–1341.
doi:10.11650/twjm/1500405511
- [9] A. Raychaudhuri, *Further results on T -coloring and frequency assignment problems*, SIAM J. Discrete Math. **7** (1994) 605–613.
doi:10.1137/S0895480189171746
- [10] D. Moser, *The star-chromatic number of planar graphs*, J. Graph Theory **24** (1997) 33–43.
doi:10.1002/(SICI)1097-0118(199701)24:1<33::AID-JGT5>3.0.CO;2-K
- [11] B. Tesman, *Applications of forbidden difference graphs to T -coloring*, Congr. Numer. **74** (1990) 15–24.
- [12] A. Vince, *Star chromatic number*, J. Graph Theory **12** (1988) 551–559.
doi:10.1002/jgt.3190120411

- [13] Y. Zhao, W. He and R. Cao, *The edge span of T -coloring on graph C_n^d* , Appl. Math. Lett. **19** (2006) 647–651.
doi:10.1016/j.aml.2005.08.016
- [14] X. Zhu, *Circular chromatic number: a survey*, Discrete Math. **229** (2001) 371–410.
doi:10.1016/S0012-365X(00)00217-X
- [15] X. Zhu, *Recent developments in circular colouring of graphs*, Algorithms Combin. **26** (2006) 497–550.
doi:10.1007/3-540-33700-8_25

Received 19 March 2018
Revised 16 October 2018
Accepted 7 December 2018