ON THE RESTRICTED SIZE RAMSEY NUMBER INVOLVING A PATH P_3

DENNY RIAMA SILABAN^{a,b,1}

EDY TRI BASKORO^a

AND

SALADIN UTTUNGGADEWA^a

^a*Combinatorial Mathematics Research Group
Faculty of Mathematics and Natural Sciences
Institut Teknologi Bandung
Jalan Ganesa 10 Bandung 40132, Indonesia*

^b*Department of Mathematics, Faculty of Mathematics
and Natural Sciences, Universitas Indonesia*

e-mail: denny@sci.ui.ac.id
ebaskoro@math.itb.ac.id
s-uttunggadewa@math.itb.ac.id

Abstract

For any pair of graphs G and H , both the size Ramsey number $\hat{r}(G, H)$ and the restricted size Ramsey number $r^*(G, H)$ are bounded above by the size of the complete graph with order equals to the Ramsey number $r(G, H)$, and bounded below by $e(G) + e(H) - 1$. Moreover, trivially, $\hat{r}(G, H) \leq r^*(G, H)$. When introducing the size Ramsey number for graph, Erdős *et al.* (1978) asked two questions; (1) Do there exist graphs G and H such that $\hat{r}(G, H)$ attains the upper bound? and (2) Do there exist graphs G and H such that $\hat{r}(G, H)$ is significantly less than the upper bound?

In this paper we consider the restricted size Ramsey number $r^*(G, H)$. We answer both questions above for $r^*(G, H)$ when $G = P_3$ and H is a connected graph.

Keywords: restricted size Ramsey number, path, connected graph, star.

2010 Mathematics Subject Classification: 05C55, 05D10.

¹Corresponding author.

REFERENCES

- [1] H. Bielak, *Remarks on the size Ramsey number of graphs*, Period. Math. Hungar. **18** (1987) 27–38.
doi:10.1007/BF01849031
- [2] H. Bielak, *Size Ramsey numbers for some regular graphs*, Electron. Notes Discrete Math. **24** (2006) 39–45.
doi:10.1016/j.endm.2006.06.007
- [3] H. Bielak, *Size Ramsey numbers for some regular graphs*, Discrete Math. **309** (2009) 6446–6449.
doi:10.1016/j.disc.2008.11.011
- [4] S.A. Burr, *A survey of noncomplete Ramsey theory for graphs*, Ann. N.Y. Acad. Sci. **328** (1979) 58–75.
doi:10.1111/j.1749-6632.1979.tb17768.x
- [5] V. Chvátal and F. Harary, *Generalized Ramsey theory for graphs, III. Small off-diagonal numbers*, Pacific J. Math. **41** (1972) 335–345.
doi:10.2140/pjm.1972.41.335
- [6] R. Diestel, *Graph Theory*, 4 Edition (Springer-Verlag Heidelberg, New York, 2005).
- [7] A. Dudek, F. Khoeini and P. Pralat, *Size-Ramsey numbers of cycles versus a path*, Discrete Math. **341** (2018) 2095–2103.
doi:10.1016/j.disc.2018.04.008
- [8] A. Dudek and P. Pralat, *An alternative proof of the linearity of the size-Ramsey number of paths*, Combin. Probab. Comput. **24** (2015) 551–555.
doi:10.1017/S096354831400056X
- [9] P. Erdős, R.J. Faudree, C.C. Rousseau and R. Schelp, *The size Ramsey number*, Period. Math. Hungar. **9** (1978) 145–161.
doi:10.1007/BF02018930
- [10] P. Erdős and R.J. Faudree, *Size Ramsey function*, in: Sets, graphs, and numbers, Colloq. Math. Soc. Janos Bolyai **60** (1991) 219–238.
- [11] R.J. Faudree, C.C. Rousseau and J. Sheehan, *A class of size Ramsey numbers involving stars*, in: Graph Theory and Combinatorics - A Volume in Honor of Paul Erdős, B. Bollobás (Ed(s)), (Academic Press, London., 1984) 273–282.
- [12] R.J. Faudree and R.H. Schelp, *A survey of results on the size Ramsey number*, in: Proc. of the Conference Paul Erdős and His Mathematics, Budapest 1999, Bolyai Soc. Math. Stud. **11** (2002) 291–309.
- [13] R.J. Faudree and J. Sheehan, *Size Ramsey numbers for small-order graphs*, J. Graph Theory **7** (1983) 53–55.
doi:10.1002/jgt.3190070107
- [14] R.J. Faudree and J. Sheehan, *Size Ramsey numbers involving stars*, Discrete Math. **46** (1983) 151–157.
doi:10.1016/0012-365X(83)90248-0

- [15] F. Harary and Z. Miller, *Generalized Ramsey theory VIII. The size Ramsey number of small graphs*, Stud. Pure Math. (1983) 271–283.
doi:10.1007/978-3-0348-5438-2_25
- [16] R. Javadi, F. Khoeini, G. Omid and A. Pokrovskiy, *On the size-Ramsey number of cycles* (2017).
arXiv:1701.07348v1 [math.CO]
- [17] S. Letzter, *Path Ramsey number for random graphs*, Combin. Probab. Comput. **25** (2016) 612–622.
doi:10.1017/S0963548315000279
- [18] R. Lortz and I. Mengersen, *Size Ramsey results for paths versus stars*, Australas. J. Combin. **18** (1998) 3–12.
- [19] M. Miralaei, G.R. Omid and M. Shahsiah, *Size Ramsey numbers of stars versus cliques* (2016).
arXiv:1601.06599v1 [math.CO]
- [20] S.P. Radziszowski, *Small Ramsey numbers*, Electron. J. Combin. (2017) #DS1.
- [21] D.R. Silaban, E.T. Baskoro and S. Uttunggadewa, *On the restricted size Ramsey number*, Procedia Computer Science **74** (2015) 21–26.
doi:10.1016/j.procs.2015.12.069
- [22] D.R. Silaban, E.T. Baskoro and S. Uttunggadewa, *Restricted size Ramsey number for P_3 versus small paths*, AIP Conf. Proc. **1707** (2016) 020020.
doi:10.1063/1.4940821
- [23] D.R. Silaban, E.T. Baskoro and S. Uttunggadewa, *Restricted size Ramsey number for path of order three versus graph of order five*, Electron. J. Graph Theory Appl. **5** (2017) 155–162.
doi:10.5614/ejgta.2017.5.1.15

Received 12 January 2018

Revised 11 October 2018

Accepted 11 October 2018