ON IMPLICIT HEAVY SUBGRAPHS AND HAMILTONICITY OF 2-CONNECTED GRAPHS

WEI ZHENG^a, WOJCIECH WIDEL^b

AND

LIGONG WANG^{a,1}

^a*Department of Applied Mathematics, School of Science
Northwestern Polytechnical University
Xi'an, Shaanxi 710072, P.R. China*

^b*Univ Rennes, INSA Rennes, CNRS, IRISA, Rennes, France*

e-mail: zhengweimath@163.com
wwidel@irisa.fr
lgwangmath@163.com

Abstract

A graph G of order n is *implicit claw-heavy* if in every induced copy of $K_{1,3}$ in G there are two non-adjacent vertices with sum of their implicit degrees at least n . We study various implicit degree conditions (including, but not limiting to, Ore- and Fan-type conditions) imposing of which on specific induced subgraphs of a 2-connected implicit claw-heavy graph ensures its Hamiltonicity. In particular, we improve a recent result of [X. Huang, *Implicit degree condition for Hamiltonicity of 2-heavy graphs*, Discrete Appl. Math. 219 (2017) 126–131] and complete the characterizations of pairs of o-heavy and f-heavy subgraphs for Hamiltonicity of 2-connected graphs.

Keywords: implicit degree, implicit o-heavy, implicit f-heavy, implicit c-heavy, Hamilton cycle.

2010 Mathematics Subject Classification: 05C45, 05C38, 05C07.

REFERENCES

- [1] P. Bedrossian, Forbidden Subgraph and Minimum Degree Conditions for Hamiltonicity, Ph.D. Thesis (Memphis State University, USA, 1991).

¹Corresponding author.

- [2] P. Bedrossian, G. Chen and R.H. Schelp, *A generalization of Fan's condition for Hamiltonicity, pancyclicity and Hamiltonian connectedness*, Discrete Math. **115** (1993) 39–50.
doi:10.1016/0012-365X(93)90476-A
- [3] J.A. Bondy and U.S.R. Murty, *Graph Theory with Applications*, GTM **244** (Springer, Berlin, 2008).
- [4] H.J. Broersma, Z. Ryjáček and I. Schiermeyer, *Dirac's minimum condition restricted to claws*, Discrete Math. **167/168** (1997) 155–166.
doi:10.1016/S0012-365X(96)00224-5
- [5] H.J. Broersma and H.J. Veldman, *Restrictions on induced subgraphs ensuring Hamiltonicity or pancyclicity of $K_{1,3}$ -free graphs*, in: Contemporary Methods in Graph Theory, (BI Wissenschaftsverlag, Mannheim, 1990) 181–194.
- [6] J. Brousek, *Forbidden triples for Hamiltonicity*, Discrete Math. **251** (2002) 71–76.
doi:10.1016/S0012-365X(01)00326-0
- [7] R. Čada, *Degree conditions on induced claws*, Discrete Math. **308** (2008) 5622–5631.
doi:10.1016/j.disc.2007.10.026
- [8] J. Cai and H. Li, *Hamilton cycles in implicit 2-heavy graphs*, Graphs Combin. **32** (2016) 1329–1337.
doi:10.1007/s00373-015-1669-4
- [9] J. Cai and Y. Zhang, *Fan-type implicit-heavy subgraphs for Hamiltonicity of implicit claw-heavy graphs*, Inform. Process. Lett. **116** (2016) 668–673.
doi:10.1016/j.ipl.2016.06.012
- [10] G. Chen, B. Wei and X. Zhang, *Forbidden graphs and Hamiltonian cycles*, preprint, 1995.
- [11] G. Chen, B. Wei and X. Zhang, *Degree-light-free graphs and Hamiltonian cycles*, Graphs Combin. **17** (2001) 409–434.
doi:10.1007/s003730170018
- [12] B. Chen and S. Zhang, *An implicit degree condition for long cycles in 2-connected graphs*, Appl. Math. Lett. **19** (2006) 1148–1151.
doi:10.1016/j.aml.2005.12.006
- [13] V. Chvátal and P. Erdős, *A note on Hamiltonian circuits*, Discrete Math. **2** (1972) 111–113.
doi:10.1016/0012-365X(72)90079-9
- [14] D. Duffus, R.J. Gould and M.S. Jacobson, *Forbidden subgraphs and the Hamiltonian theme*, Proceedings of the 4th International Conference on the Theory and Applications of Graphs, Kalamazoo, 1980 (Wiley, New York, 1981) 297–316.
- [15] G. Fan, *New sufficient conditions for cycles in graphs*, J. Combin. Theory Ser. B **37** (1984) 221–227.
doi:10.1016/0095-8956(84)90054-6

- [16] R. Faudree, Z. Ryjáček and I. Schiermeyer, *Forbidden subgraphs and cycle extendability*, J. Combin. Math. Combin. Comput. **19** (1995) 109–128.
- [17] R.J. Faudree and R.J. Gould, *Characterizing forbidden pairs for Hamiltonian properties*, Discrete Math. **173** (1997) 45–60.
doi:10.1016/S0012-365X(96)00147-1
- [18] S. Goodman and S.T. Hedetniemi, *Sufficient conditions for a graph to be Hamiltonian*, J. Combin. Theory Ser. B **16** (1974) 175–180.
doi:10.1016/0095-8956(74)90061-6
- [19] R.J. Gould and M.S. Jacobson, *Forbidden subgraphs and Hamiltonian properties of graphs*, Discrete Math. **42** (1982) 189–196.
doi:10.1016/0012-365X(82)90216-3
- [20] Z. Hu, *A generalization of fan's condition and forbidden subgraph conditions for Hamiltonicity*, Discrete Math. **196** (1999) 167–175.
doi:10.1016/S0012-365X(98)00200-3
- [21] X. Huang, *Hamilton cycles in implicit claw-heavy graphs*, Inform. Process. Lett. **114** (2014) 676–679.
doi:10.1016/j.ipl.2014.06.007
- [22] X. Huang, *Implicit degree condition for Hamiltonicity of 2-heavy graphs*, Discrete Appl. Math. **219** (2017) 126–131.
doi:10.1016/j.dam.2016.10.031
- [23] B. Li and B. Ning, *Heavy subgraphs, stability and Hamiltonicity*, Discuss. Math. Graph Theory **37** (2017) 691–710.
doi:10.7151/dmgt.1967
- [24] B. Li, B. Ning and S. Zhang, *Degree conditions restricted to induced paths for Hamiltonicity of claw-heavy graphs*, Acta Math. Sin. (Engl. Ser.) **33** (2017) 301–310.
doi:10.1007/s10114-016-5735-5
- [25] B. Li, Z. Ryjáček, Y. Wang and S. Zhang, *Pairs of heavy subgraphs for Hamiltonicity of 2-connected graphs*, SIAM J. Discrete Math. **26** (2012) 1088–1103.
doi:10.1137/11084786X
- [26] B. Li and P. Vrána, *Forbidden pairs of disconnected graphs implying Hamiltonicity*, J. Graph Theory **84** (2017) 249–261.
doi:10.1002/jgt.22024
- [27] G. Li, B. Wei and T. Gao, *A structural method for Hamiltonian graphs*, Australas. J. Combin. **11** (1995) 257–262.
- [28] H. Li, W. Ning and J. Cai, *An implicit degree condition for cyclability in graphs*, Lecture Notes in Comput. Sci. **6681** (2011) 82–89.
doi:10.1007/978-3-642-21204-8_12
- [29] B. Ning, *Fan-type degree condition restricted to triples of induced subgraphs ensuring Hamiltonicity*, Inform. Process. Lett. **113** (2013) 823–826.
doi:10.1016/j.ipl.2013.07.014

- [30] B. Ning and S. Zhang, *Ore- and Fan-type heavy subgraphs for Hamiltonicity of 2-connected graphs*, Discrete Math. **313** (2013) 1715–1725.
doi:10.1016/j.disc.2013.04.023
- [31] O. Ore, *Note on Hamilton circuits*, Amer. Math. Monthly **67** (1960) 55.
doi:10.2307/2308928
- [32] Y. Zhu, H. Li and X. Deng, *Implicit-degrees and circumferences*, Graphs Combin. **5** (1989) 283–290.
doi:10.1007/BF01788680

Received 29 May 2017
Revised 20 August 2018
Accepted 29 August 2018