DOMINATION NUMBER, INDEPENDENT DOMINATION NUMBER AND 2-INDEPENDENCE NUMBER IN TREES

NASRIN DEHGARDI¹

Department of Mathematics and Computer Science
Sirjan University of Technology, Sirjan, I.R. Iran

e-mail: n.dehgardi@sirjantech.ac.ir

SEYED MAHMOUD SHEIKHOESLAMI, MINA VALINAVAZ

Department of Mathematics
Azarbaijan Shahid Madani University, Tabriz, I.R. Iran

e-mail: s.m.sheikholeslami
m.valinavaz@azaruniv.ac.ir

HAMIDEH ARAM

Department of Mathematics
Gareziaeddin Center, Khoy Branch
Islamic Azad University, Khoy, Iran

e-mail: hamideh.aram@gmail.com

AND

LUTZ VOLKMANN

Lehrstuhl II für Mathematik
RWTH Aachen University, 52056 Aachen, Germany

e-mail: volkm@math2.rwth-aachen.de

Abstract

For a graph G , let $\gamma(G)$ be the domination number, $i(G)$ be the independent domination number and $\beta_2(G)$ be the 2-independence number. In this paper, we prove that for any tree T of order $n \geq 2$, $4\beta_2(T) - 3\gamma(T) \geq 3i(T)$, and we characterize all trees attaining equality. Also we prove that for every tree T of order $n \geq 2$, $i(T) \leq \frac{3\beta_2(T)}{4}$, and we characterize all extreme trees.

¹Corresponding author.

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