LOW 5-STARs AT 5-VERTICES IN 3-POLYTOPES WITH MINIMUM DEGREE 5 AND NO VERTICES OF DEGREE FROM 7 TO 9¹

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Abstract

In 1940, Lebesgue gave an approximate description of the neighborhoods of 5-vertices in the class $\mathbf{P}_5$ of 3-polytopes with minimum degree 5.

Given a 3-polytope P , by $h_5(P)$ we denote the minimum of the maximum degrees (height) of the neighborhoods of 5-vertices (minor 5-stars) in P .

Recently, Borodin, Ivanova and Jensen showed that if a polytope P in $\mathbf{P}_5$ is allowed to have a 5-vertex adjacent to two 5-vertices and two more vertices of degree at most 6, called a $(5, 5, 6, 6, \infty)$ -vertex, then $h_5(P)$ can be arbitrarily large. Therefore, we consider the subclass $\mathbf{P}_5^*$ of 3-polytopes in $\mathbf{P}_5$ that avoid $(5, 5, 6, 6, \infty)$ -vertices.

For each P^* in $\mathbf{P}_5^*$ without vertices of degree from 7 to 9, it follows from Lebesgue's Theorem that $h_5(P^*) \leq 17$. Recently, this bound was lowered by Borodin, Ivanova, and Kazak to the sharp bound $h_5(P^*) \leq 15$ assuming the absence of vertices of degree from 7 to 11 in P^* .

In this note, we extend the bound $h_5(P^*) \leq 15$ to all P^* s without vertices of degree from 7 to 9.

Keywords: planar map, planar graph, 3-polytope, structural properties, 5-star, weight, height.

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