STRONG TUTTE TYPE CONDITIONS AND FACTORS OF GRAPHS

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Abstract

Let $\text{odd}(G)$ denote the number of odd components of a graph G and $k \geq 2$ be an integer. We give sufficient conditions using $\text{odd}(G - S)$ for a graph G to have an even factor. Moreover, we show that if a graph G satisfies $\text{odd}(G - S) \leq \max\{1, (1/k)|S|\}$ for all $S \subset V(G)$, then G has a $(k - 1)$ -regular factor for $k \geq 3$ or an **H**-factor for $k = 2$, where we say that G has an **H**-factor if for every labeling $h : V(G) \rightarrow \{\text{red}, \text{blue}\}$ with $\#\{v \in V(G) : f(v) = \text{red}\}$ even, G has a spanning subgraph F such that $\deg_F(x) = 1$ if $h(x) = \text{red}$ and $\deg_F(x) \in \{0, 2\}$ otherwise.

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