2-SPANNING CYCLABILITY PROBLEMS OF SOME GENERALIZED PETERSEN GRAPHS

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Abstract

A graph G is called r -spanning cyclable if for every r distinct vertices $v_1, v_2, \dots, v_r$ of G , there exists r cycles $C_1, C_2, \dots, C_r$ in G such that v_i is on C_i for every i , and every vertex of G is on exactly one cycle C_i . In this paper, we consider the 2-spanning cyclable problem for the generalized Petersen graph $GP(n, k)$. We solved the problem for $k \leq 4$. In addition, we provide an additional observation for general k as well as stating a conjecture.

Keywords: Petersen graph, spanning cyclable.

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