REGULAR COLORINGS IN REGULAR GRAPHS

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Abstract

An $(r - 1, 1)$ -coloring of an r -regular graph G is an edge coloring (with arbitrarily many colors) such that each vertex is incident to $r - 1$ edges of one color and 1 edge of a different color. In this paper, we completely characterize all 4-regular pseudographs (graphs that may contain parallel edges and loops) which do not have a $(3, 1)$ -coloring. Also, for each $r \geq 6$ we construct graphs that are not $(r - 1, 1)$ -colorable and, more generally, are not $(r - t, t)$ -colorable for small t .

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