EXTREMAL GRAPHS FOR A BOUND ON THE ROMAN DOMINATION NUMBER

AHMED BOUCHOU¹, MOSTAFA BLIDIA²

AND

MUSTAPHA CHELLALI²

¹*Department of Mathematics and Computer Science
University of Médéa, Algeria*

²*LAMDA-RO Laboratory
Department of Mathematics
University of Blida B.P. 270, Blida, Algeria*

e-mail: bouchou.ahmed@yahoo.fr
m_blidia@yahoo.fr
m_chellali@yahoo.com

Abstract

A Roman dominating function on a graph $G = (V, E)$ is a function $f: V(G) \rightarrow \{0, 1, 2\}$ such that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v with $f(v) = 2$. The weight of a Roman dominating function is the value $w(f) = \sum_{u \in V(G)} f(u)$. The minimum weight of a Roman dominating function on a graph G is called the Roman domination number of G , denoted by $\gamma_R(G)$. In 2009, Chambers, Kinnerley, Prince and West proved that for any graph G with n vertices and maximum degree Δ , $\gamma_R(G) \leq n + 1 - \Delta$. In this paper, we give a characterization of graphs attaining the previous bound including trees, regular and semiregular graphs. Moreover, we prove that the problem of deciding whether $\gamma_R(G) = n + 1 - \Delta$ is *co-NP*-complete. Finally, we provide a characterization of extremal graphs of a Nordhaus–Gaddum bound for $\gamma_R(G) + \gamma_R(\overline{G})$, where $\overline{G}$ is the complement graph of G .

Keywords: Roman domination, Roman domination number, Nordhaus–Gaddum inequalities.

2010 Mathematics Subject Classification: 05C69.

REFERENCES

- [1] S. Bermudo, H. Fernau and J. Sigarreta, *The differential and the Roman domination number of a graph*, Appl. Anal. Discrete Math. **8** (2014) 155–171.
doi:10.2298/AADM140210003B
- [2] E.W. Chambers, B. Kinnersley, N. Prince and D.B. West, *Extremal problems for Roman domination*, SIAM J. Discrete Math. **23** (2009) 1575–1586.
doi:10.1137/070699688
- [3] E.J. Cockayne, P.A. Dreyer Jr., S.M. Hedetniemi and S.T. Hedetniemi, *Roman domination in graphs*, Discrete Math. **278** (2004) 11–22.
doi:10.1016/j.disc.2003.06.004
- [4] M.R. Garey and D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-completeness* (Freeman, New York, 1979).
- [5] W. McCuaig and B. Shepherd, *Domination in graphs with minimum degree two*, J. Graph Theory **13** (1989) 749–762.
doi:10.1002/jgt.3190130610
- [6] B.P. Mobaraky and S.M. Sheikholeslami, *Bounds on Roman domination numbers of graphs*, Mat. Vesnik **60** (2008) 247–253.
- [7] C.S. ReVelle and K.E. Rosing, *Defendens Imperium Romanum: a classical problem in military strategy*, Amer. Math. Monthly **107** (2000) 585–594.
doi:10.1080/00029890.2000.12005243
- [8] I. Stewart, *Defend the Roman Empire!*, Sci. Amer. **281** (1999) 136–138.
doi:10.1038/scientificamerican1299-136

Received 11 December 2017

Revised 24 April 2018

Accepted 24 April 2018