FACIAL $[r, s, t]$ -COLORINGS OF PLANE GRAPHS

JÚLIUS CZAP, PETER ŠUGEREK

Department of Applied Mathematics and Business Informatics
Faculty of Economics, Technical University of Košice
Němcovej 32, 040 01 Košice, Slovakia

e-mail: julius.czap@tuke.sk
peter.sugerek@tuke.sk

STANISLAV JENDROĽ

AND

JURAJ VALISKA

Institute of Mathematics, P.J. Šafárik University
Jesenná 5, 040 01 Košice, Slovakia

e-mail: stanislav.jendrol@upjs.sk
juraj.valiska@student.upjs.sk

Abstract

Let G be a plane graph. Two edges are facially adjacent in G if they are consecutive edges on the boundary walk of a face of G . Given nonnegative integers r, s , and t , a facial $[r, s, t]$ -coloring of a plane graph $G = (V, E)$ is a mapping $f : V \cup E \rightarrow \{1, \dots, k\}$ such that $|f(v_1) - f(v_2)| \geq r$ for every two adjacent vertices v_1 and v_2 , $|f(e_1) - f(e_2)| \geq s$ for every two facially adjacent edges e_1 and e_2 , and $|f(v) - f(e)| \geq t$ for all pairs of incident vertices v and edges e . The facial $[r, s, t]$ -chromatic number $\bar{\chi}_{r,s,t}(G)$ of G is defined to be the minimum k such that G admits a facial $[r, s, t]$ -coloring with colors $1, \dots, k$. In this paper we show that $\bar{\chi}_{r,s,t}(G) \leq 3r + 3s + t + 1$ for every plane graph G . For some triplets $[r, s, t]$ and for some families of plane graphs this bound is improved. Special attention is devoted to the cases when the parameters r, s , and t are small.

Keywords: plane graph, boundary walk, edge-coloring, vertex-coloring, total-coloring.

2010 Mathematics Subject Classification: 05C10, 05C15.

REFERENCES

- [1] K. Appel and W. Haken, *Every planar map is four colorable, Part I: Discharging*, Illinois J. Math. **21** (1977) 429–490.
- [2] K. Appel, W. Haken and J. Koch, *Every planar map is four colorable, Part II: Reducibility*, Illinois J. Math. **21** (1977) 491–567.
- [3] J.A. Bondy and U.S.R. Murty, *Graph Theory* (Springer, New York, 2008).
- [4] D. Chen and W. Wang, *$(2, 1)$ -total labelling of outerplanar graphs*, Discrete Appl. Math. **155** (2007) 2585–2593.
doi:10.1016/j.dam.2007.07.016
- [5] J. Czap and S. Jendrol', *Facially-constrained colorings of plane graphs: A survey*, Discrete Math. **340** (2017) 2691–2703.
doi:10.1016/j.disc.2016.07.026
- [6] J. Czap and P. Šugerek, *Facial total-coloring of bipartite plane graphs*, Int. J. Pure Appl. Math. **115** (2017) 115–121.
doi:10.12732/ijpam.v115i1.9
- [7] L. Dekar, B. Effantin and H. Kheddouci, *$[r, s, t]$ -colorings of graph products*, Graphs Combin. **30** (2014) 1135–1147.
doi:10.1007/s00373-013-1338-4
- [8] L. Dekar, B. Effantin and H. Kheddouci, *$[r, s, t]$ -colorings of trees and bipartite graphs*, Discrete Math. **310** (2010) 260–269.
doi:10.1016/j.disc.2008.09.021
- [9] I. Fabrici, S. Jendrol' and M. Vrbjarová, *Unique-maximum edge-colouring of plane graphs with respect to faces*, Discrete Appl. Math. **185** (2015) 239–243.
doi:10.1016/j.dam.2014.12.002
- [10] I. Fabrici, S. Jendrol' and M. Vrbjarová, *Facial entire colourings of plane graphs*, Discrete Math. **339** (2016) 626–631.
doi:10.1016/j.disc.2015.09.011
- [11] I. Fabrici, S. Jendrol' and M. Voigt, *Facial list colourings of plane graphs*, Discrete Math. **339** (2016) 2826–2831.
doi:10.1016/j.disc.2016.05.034
- [12] H. Grötzsch, *Ein Dreifarbensatz für dreikreisfreie Netze auf der Kugel*, Wiss. Z. Martin-Luther-Universität, Halle-Wittenberg, Math.-Nat. Reihe **8** (1959) 109–120.
- [13] T. Hasunuma, T. Ishii, H. Ono and Y. Uno, *A tight upper bound on the $(2, 1)$ -total labeling number of outerplanar graphs*, J. Discrete Algorithms **14** (2012) 189–206.
doi:10.1016/j.jda.2011.12.020
- [14] F. Havet and S. Thomassé, *Complexity of $(p, 1)$ -total labelling*, Discrete Appl. Math. **157** (2009) 2859–2870.
doi:10.1016/j.dam.2009.03.021

- [15] F. Havet and M.-L. Yu, $(p, 1)$ -total labelling of graphs, Discrete Math. **308** (2008) 496–513.
doi:10.1016/j.disc.2007.03.034
- [16] A. Kemnitz and J. Lehmann, $[r, s, t]$ -colorings of stars, Congr. Numer. **185** (2007) 65–80.
- [17] A. Kemnitz, J. Lehmann and M. Marangio, $[1, 1, 2]$ -colorings of complete graphs, Congr. Numer. **208** (2011) 99–113.
- [18] A. Kemnitz and M. Marangio, $[r, s, t]$ -colorings of graphs, Discrete Math. **307** (2007) 199–207.
doi:10.1016/j.disc.2006.06.030
- [19] A. Kemnitz, M. Marangio and P. Mihók, $[r, s, t]$ -chromatic numbers and hereditary properties of graphs, Discrete Math. **307** (2007) 916–922.
doi:10.1016/j.disc.2005.11.055
- [20] A. Kemnitz, M. Marangio and Zs. Tuza, $[1, 1, t]$ -colorings of complete graphs, Graphs Combin. **29** (2013) 1041–1050.
doi:10.1007/s00373-012-1153-3
- [21] W. Liao and M. Li, $[r, s, t]$ -colorings of friendship graphs and wheels, Graphs Combin. **31** (2015) 2275–2292.
doi:10.1007/s00373-014-1502-5
- [22] I. Schiermeyer and M.S. Villà, $[r, s, t]$ -colourings of paths, Opuscula Math. **27** (2007) 131–149.
- [23] G. Sethuraman and A. Velankanni, $(2, 1)$ -total labeling of a class of subcubic graphs, Electron. Notes Discrete Math. **48** (2015) 259–266.
doi:10.1016/j.endm.2015.05.039
- [24] P.G. Tait, *Remarks on the colouring of maps*, Proc. Roy. Soc. Edinburgh **10** (1880) 729.
- [25] M.S. Villà, $[r, s, t]$ -Colourings of Paths, Cycles and Stars, Doctoral Thesis (TU Bergakademie, Freiberg, 2005).
- [26] W. Wang and D. Chen, $(2, 1)$ -total number of trees with maximum degree three, Inform. Process. Lett. **109** (2009) 805–810.
doi:10.1016/j.ipl.2009.03.027

Received 30 October 2017

Revised 9 March 2018

Accepted 9 March 2018