A NOTE ON CYCLES IN LOCALLY HAMILTONIAN AND LOCALLY HAMILTON-CONNECTED GRAPHS

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Abstract

Let $\mathcal{P}$ be a property of a graph. A graph G is said to be locally $\mathcal{P}$, if the subgraph induced by the open neighbourhood of every vertex in G has property $\mathcal{P}$. Ryjáček conjectures that every connected, locally connected graph is weakly pancyclic. Motivated by the above conjecture, van Aardt *et al.* [S.A.van Aardt, M. Frick, O.R. Oellermann and J.P.de Wet, *Global cycle properties in locally connected, locally traceable and locally Hamiltonian graphs*, Discrete Appl. Math. 205 (2016) 171–179] investigated the global cycle structures in connected, locally traceable/Hamiltonian graphs. Among other results, they proved that a connected, locally Hamiltonian graph G with maximum degree at least $|V(G)| - 5$ is weakly pancyclic. In this note, we improve this result by showing that such a graph with maximum degree at least $|V(G)| - 6$ is weakly pancyclic. Furthermore, we show that a connected, locally Hamilton-connected graph with maximum degree at most 7 is fully cycle extendable.

Keywords: locally connected, locally Hamiltonian, locally Hamilton-connected, fully cycle extendability, weakly pancyclicity.

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REFERENCES

- [1] A. Adamaszek, M. Adamaszek, M. Mnich and J.M. Schmidt, *Lower bounds for locally highly connected graphs*, Graphs Combin. **32** (2016) 1641–1650.
doi:10.1007/s00373-016-1686-y
- [2] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer, New York, 2008).
- [3] A. Borchert, S. Nicol and O.R. Oellermann, *Global cycle properties of locally isometric graphs*, Discrete Appl. Math. **205** (2016) 16–26.
doi:10.1016/j.dam.2016.01.026
- [4] C. Brause, D. Rautenbach and I. Schiermeyer, *Local connectivity, local degree conditions, some forbidden induced subgraphs, and cycle extendability*, Discrete Math. **340** (2017) 596–606.
doi:10.1016/j.disc.2016.11.035
- [5] G. Chartrand and R.E. Pippert, *Locally connected graphs*, Časopis Pěst. Mat. **99** (1974) 158–163.
- [6] J.P. de Wet and S.A. van Aardt, *Traceability of locally Hamiltonian and locally traceable graphs*, Discrete Math. Theor. Comput. Sci. **17** (2016) 245–262.
- [7] J.P. de Wet, M. Frick and S.A. van Aardt, *Hamiltonicity of locally Hamiltonian and locally traceable graphs*, Discrete Appl. Math. **236** (2018) 137–152.
doi:10.1016/j.dam.2017.10.030
- [8] G.R.T. Hendry, *Extending cycles in graphs*, Discrete Math. **85** (1990) 59–72.
doi:10.1016/0012-365X(90)90163-C
- [9] D.J. Oberly and D.P. Sumner, *Every connected, locally connected nontrivial graph with no induced claw is Hamiltonian*, J. Graph Theory **3** (1979) 351–356.
doi:10.1002/jgt.3190030405
- [10] C.M. Pareek, *On the maximum degree of locally Hamiltonian non-Hamiltonian graphs*, Util. Math. **23** (1983) 103–120.
- [11] C.M. Pareek and Z. Skupień, *On the smallest non-Hamiltonian locally Hamiltonian graph*, J. Univ. Kuwait Sci. **10** (1983) 9–16.
- [12] Z. Skupień, *On the locally Hamiltonian graphs and Kuratowski's theorem*, Bull. Acad. Pol. Sci., Ser. Sci. Math. Astron. Phys. **13** (1965) 615–619.
- [13] Z. Skupień, *Locally Hamiltonian and planar graphs*, Fund. Math. **58** (1966) 193–200.
doi:10.4064/fm-58-2-193-200
- [14] S.A. van Aardt, M. Frick, O.R. Oellermann and J.P. de Wet, *Global cycle properties in locally connected, locally traceable and locally Hamiltonian graphs*, Discrete Appl. Math. **205** (2016) 171–179.
doi:10.1016/j.dam.2015.09.022
- [15] D.B. West, *Research problems*, Discrete Math. **272** (2003) 301–306.
doi:10.1016/S0012-365X(03)00207-3

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