THE PROPER DIAMETER OF A GRAPH

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Abstract

A proper edge-coloring of a graph is a coloring in which adjacent edges receive distinct colors. A path is properly colored if consecutive edges have distinct colors, and an edge-colored graph is properly connected if there exists a properly colored path between every pair of vertices. In such a graph, we introduce the notion of the graph's proper diameter—which is a function of both the graph and the coloring—and define it to be the maximum length of a shortest properly colored path between any two vertices in the graph. We consider various families of graphs to find bounds on the gap between the diameter and possible proper diameters, paying singular attention to 2-colorings.

Keywords: diameter, properly connected, proper diameter.

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REFERENCES

- [1] E. Andrews, E. Laforge, C. Lumduanhom and P. Zhang, *On proper-path colorings in graphs*, J. Combin. Math. Combin. Comput. **97** (2016) 189–207.
- [2] V. Borozan, S. Fujita, A. Gerek, C. Magnant, Y. Manoussakis, L. Montero and Zs. Tuza, *Proper connection of graphs*, Discrete Math. **312** (2012) 2550–2560.
doi:10.1016/j.disc.2011.09.003
- [3] G. Chartrand, G. L. Johns, A. K. McKeon and P. Zhang, *Rainbow connection in graphs*, Math. Bohem. **133** (2008) 85–98.
- [4] G. Chartrand, G. Johns, K. McKeon and P. Zhang, *The rainbow connectivity of a graph*, Networks **54** (2009) 75–81.
doi:10.1002/net.20296
- [5] S. Fujita, A. Gerek and C. Magnant, *Proper connection with many colors*, J. Comb. **3** (2012) 683–693.
doi:10.4310/JOC.2012.v3.n4.a6
- [6] E. Laforge, C. Lumduanhom and P. Zhang, *Characterizations of graphs having large proper connection numbers*, Discuss. Math. Graph Theory **36** (2016) 439–453.
doi:10.7151/dmgt.1867
- [7] E. Laforge, C. Lumduanhom and P. Zhang, *Chromatic-connection in graphs*, Congr. Numer. **225** (2015) 37–54.
- [8] X. Li and C. Magnant, *Properly colored notions of connectivity—a dynamic survey*, Theory Appl. Graphs **0** (2015), Iss. 1, Article 2.
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