DOMINATION SUBDIVISION AND DOMINATION MULTISUBDIVISION NUMBERS OF GRAPHS

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Abstract

The *domination subdivision number* $sd(G)$ of a graph G is the minimum number of edges that must be subdivided (where an edge can be subdivided at most once) in order to increase the domination number of G . It has been shown [10] that $sd(T) \leq 3$ for any tree T . We prove that the decision problem of the domination subdivision number is NP-complete even for bipartite graphs. For this reason we define the *domination multisubdivision number* of a nonempty graph G as a minimum positive integer k such that there exists an edge which must be subdivided k times to increase the domination number of G . We show that $msd(G) \leq 3$ for any graph G . The domination subdivision number and the domination multisubdivision number of a graph are incomparable in general, but we show that for trees these two parameters are equal. We also determine the domination multisubdivision number for some classes of graphs.

Keywords: domination, domination subdivision number, domination multisubdivision number, trees, computational complexity.

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REFERENCES

- [1] H. Aram, S.M. Sheikholeslami and O. Favaron, *Domination subdivision numbers of trees*, Discrete Math. **309** (2009) 622–628.
doi:10.1016/j.disc.2007.12.085
- [2] D. Avella-Alaminos, M. Dettlaff, M. Lemańska and R. Zuazua, *Total domination multisubdivision number of a graph*, Discuss. Math. Graph Theory **35** (2015) 315–327.
doi:10.7151/dmgt.1798
- [3] S. Benecke and C.M. Mynhardt, *Trees with domination subdivision number one*, Australas. J. Combin. **42** (2008) 201–209.
- [4] A. Bhattacharya and G.R. Vijayakumar, *Effect of edge-subdivision on vertex-domination in a graph*, Discuss. Math. Graph Theory **22** (2002) 335–347.
doi:10.7151/dmgt.1179
- [5] O. Favaron, T.W. Haynes and S.T. Hedetniemi, *Domination subdivision numbers in graphs*, Util. Math. **66** (2004) 195–209.
- [6] O. Favaron, H. Karami and S.M. Sheikholeslami, *Disproof of a conjecture on the subdivision domination number of a graph*, Graphs Combin. **24** (2008) 309–312.
doi:10.1007/s00373-008-0788-6
- [7] M.R. Garey and D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness* (Freeman, San Francisco, 1979).
- [8] T.W. Haynes, S.M. Hedetniemi and S.T. Hedetniemi, *Domination and independence subdivision numbers of graphs*, Discuss. Math. Graph Theory **20** (2000) 271–280.
doi:10.7151/dmgt.1126
- [9] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs* (Marcel Dekker Inc., New York, 1998).
- [10] S. Velammal, *Studies in Graph Theory: Covering, Independence, Domination and Related Topics*, Ph.D.Thesis (Manonmaniam Sundaranar University, Tirunelveli, 1997).

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