1-RESTRICTED OPTIMAL RUBBLING ON GRAPHS

ROBERT A. BEELER

Department of Mathematics and Statistics
East Tennessee State University
Johnson City, TN 37614-1700 USA

e-mail: beelerr@etsu.edu

TERESA W. HAYNES

Department of Mathematics and Statistics
East Tennessee State University
Johnson City, TN 37614-1700 USA
Department of Mathematics
University of Johannesburg
Auckland Park, South Africa

e-mail: haynes@etsu.edu

AND

KYLE MURPHY

Department of Mathematics
Iowa State University
Ames, IA 50011 USA

e-mail: kylem2@iastate.edu

Abstract

Let G be a graph with vertex set V and a distribution of pebbles on the vertices of V . A pebbling move consists of removing two pebbles from a vertex and placing one pebble on a neighboring vertex, and a rubbing move consists of removing a pebble from each of two neighbors of a vertex v and placing a pebble on v . We seek an initial placement of a minimum total number of pebbles on the vertices in V , so that no vertex receives more than one pebble and for any given vertex $v \in V$, it is possible, by a sequence of pebbling and rubbing moves, to move at least one pebble to v . This minimum number of pebbles is the 1-restricted optimal rubbing number. We determine the 1-restricted optimal rubbing numbers for Cartesian products. We also present bounds on the 1-restricted optimal rubbing number.

Keywords: graph pebbling, graph rubbing, optimal rubbing, t -restricted optimal pebbling.

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