ON THE TOTAL ROMAN DOMINATION IN TREES

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Abstract

A *total Roman dominating function* on a graph G is a function $f : V(G) \rightarrow \{0, 1, 2\}$ satisfying the following conditions: (i) every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$ and (ii) the subgraph of G induced by the set of all vertices of positive weight has no isolated vertex. The weight of a total Roman dominating function f is the value $f(V(G)) = \sum_{u \in V(G)} f(u)$. The *total Roman domination number* $\gamma_{tR}(G)$ is the minimum weight of a total Roman dominating function of G . Ahangar *et al.* in [H.A. Ahangar, M.A. Henning, V. Samodivkin and I.G. Yero, *Total Roman domination in graphs*, Appl. Anal. Discrete Math. 10 (2016) 501–517] recently showed that for any graph G without isolated vertices, $2\gamma(G) \leq \gamma_{tR}(G) \leq 3\gamma(G)$, where $\gamma(G)$ is the domination number of G , and they raised the problem of characterizing the graphs G achieving these upper and lower bounds. In this paper, we provide a constructive characterization of these trees.

Keywords: total Roman dominating function, total Roman domination number, trees.

2010 Mathematics Subject Classification: 05C69.

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Received 14 August 2017
Revised 26 September 2017
Accepted 26 September 2017