$L(2, 1)$ -LABELING OF CIRCULANT GRAPHS

SARBARI MITRA AND SOUMYA BHOUMIK

*Department of Mathematics
Fort Hays State University, Hays, USA*

e-mail: s_mitra@fhsu.edu
s_bhoumik@fhsu.edu

Abstract

An $L(2, 1)$ -labeling of a graph Γ is an assignment of non-negative integers to the vertices such that adjacent vertices receive labels that differ by at least 2, and those at a distance of two receive labels that differ by at least one. Let $\lambda_2^1(\Gamma)$ denote the least λ such that Γ admits an $L(2, 1)$ -labeling using labels from $\{0, 1, \dots, \lambda\}$. A Cayley graph of group G is called a circulant graph of order n , if $G = \mathbb{Z}_n$. In this paper initially we investigate the upper bound for the span of the $L(2, 1)$ -labeling for Cayley graphs on cyclic groups with “large” connection sets. Then we extend our observation and find the span of $L(2, 1)$ -labeling for any circulants of order n .

Keywords: graph coloring, $L(2, 1)$ -labeling, circulants.

2010 Mathematics Subject Classification: 05C78.

REFERENCES

- [1] T. Araki, *Labeling bipartite permutation graphs with a condition at distance two*, Discrete Appl. Math. **157** (2009) 1677–1686.
doi:10.1016/j.dam.2009.02.004
- [2] P. Bahls, *Channel assignment on Cayley graphs*, J. Graph Theory **67** (2011) 169–177.
doi:10.1002/jgt.20523
- [3] H.L. Bodlaender, T. Kloks, R.B. Tan and J. van Leeuwen, *Approximations for λ -colorings of graphs*, Comput. J. **47** (2004) 193–204.
doi:10.1093/comjnl/47.2.193
- [4] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer, New York, 2008).
- [5] T. Calamoneri, S. Caminiti, S. Olariu and R. Petreschi, *On the $L(h, k)$ -labeling of co-comparability graphs and circular-arc graphs*, Networks **53** (2009) 27–34.
doi:10.1002/net.20257

- [6] T. Calamoneri, *The $L(h,k)$ -labelling problem: An updated survey and annotated bibliography*, Comput. J. **54** (2011) 1344–1371.
doi:10.1093/comjnl/bxr037
- [7] T. Calamoneri, *The $L(h,k)$ -labelling problem: An updated survey and annotated bibliography*, (2014).
<http://wwwusers.di.uniroma1.it/~calamo/PDF-FILES/survey.pdf>
- [8] G.J. Chang and D. Kuo, *The $L(2,1)$ -labeling problem on graphs*, SIAM J. Discrete Math. **9** (1996) 309–316.
doi:10.1137/S0895480193245339
- [9] J. Griggs and R.K. Yeh, *Labelling graphs with a condition at distance 2*, SIAM J. Discrete Math. **5** (1992) 586–595.
doi:10.1137/0405048
- [10] T. Hasunuma, T. Ishii, H. Ono and Y. Uno, *A linear time algorithm for $L(2,1)$ -labeling of trees*, Lect. Notes Comput. Sci. **5757** (2009) 35–46.
doi:10.1007/978-3-642-04128-0_4
- [11] F. Havet, B. Reed and J.S. Sereni, *$L(2,1)$ -labelling of graphs*, in: Proc. 19th Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2008, SIAM (2008) 621–630.
- [12] Y.Z. Huang, C.Y. Chiang, L.H. Huang and H.G. Yeh, *On $L(2,1)$ -labeling of generalized Petersen graphs*, J. Comb. Optim. **24** (2012) 266–279.
doi:10.1007/s10878-011-9380-8
- [13] X. Li, V. Mak-Hau and S. Zhou, *The $L(2,1)$ -labelling problem for cubic Cayley graphs on dihedral groups*, J. Comb. Optim. **25** (2013) 716–736.
doi:10.1007/s10878-012-9525-4
- [14] S. Panda and P. Goel, *$L(2,1)$ -labeling of dually chordal graphs and strongly orderable graphs*, Inform. Process. Lett. **112** (2012) 552–556.
doi:10.1016/j.ipl.2012.04.003
- [15] F.S. Roberts, private communication through J. Griggs, (1988).
- [16] D. Sakai, *Labeling chordal graphs: Distance two condition*, SIAM J. Discrete Math. **7** (1994) 133–140.
doi:10.1137/S0895480191223178
- [17] Z. Shao and R.K. Yeh, *The $L(2,1)$ -labeling and operations of graphs*, IEEE Trans. Circuits Syst. I: Regul. Pap. **52** (2005) 668–671.
doi:10.1109/TCSI.2004.840484
- [18] R.K. Yeh, *A survey on labeling graphs with a condition at distance two*, Discrete Math. **306** (2006) 1217–1231.
doi:10.1016/j.disc.2005.11.029
- [19] S. Zhou, *Labeling Cayley graphs on Abelian groups*, SIAM J. Discrete Math. **19** (2006) 985–1003.
doi:10.1137/S0895480102404458

Received 6 December 2016

Revised 22 June 2017

Accepted 22 June 2017