TOTAL COLORINGS OF EMBEDDED GRAPHS WITH NO 3-CYCLES ADJACENT TO 4-CYCLES

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Abstract

A *total- k -coloring* of a graph G is a coloring of $V \cup E$ using k colors such that no two adjacent or incident elements receive the same color. The *total chromatic number* $\chi''(G)$ of G is the smallest integer k such that G has a total- k -coloring. Let G be a graph embedded in a surface of Euler characteristic $\varepsilon \geq 0$. If G contains no 3-cycles adjacent to 4-cycles, that is, no 3-cycle has a common edge with a 4-cycle, then $\chi''(G) \leq \max\{8, \Delta + 1\}$.

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