ANTIPODAL EDGE-COLORINGS OF HYPERCUBES

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Abstract

Two vertices of the k -dimensional hypercube Q_k are *antipodal* if they differ in every coordinate. Edges uv and xy are *antipodal* if u is antipodal to x and v is antipodal to y . An *antipodal edge-coloring* of Q_k is a 2-edge-coloring such that antipodal edges always have different colors. Norine conjectured that for $k \geq 2$, in every antipodal edge-coloring of Q_k some two antipodal vertices are connected by a monochromatic path. Feder and Subi proved this for $k \leq 5$. We prove it for $k \leq 6$.

Keywords: antipodal edge-coloring, hypercube, monochromatic geodesic.

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REFERENCES

- [1] M. DeVos and S. Norine, *Edge-antipodal colorings of cubes*, The Open Problem Garden.
http://garden.irmacs.sfu.ca/?q=op/edge_antipodal_colorings_of_cubes
- [2] T. Feder and C. Subi, *On hypercube labellings and antipodal monochromatic paths*, Discrete Appl. Math. **161** (2013) 1421–1426.
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- [3] K. Gandhi, *Maximal monochromatic geodesics in an antipodal coloring of hypercube* (2015), manuscript.
<http://math.mit.edu/research/highschool/primes/materials/2014/Gandhi.pdf>
- [4] I. Leader and E. Long, *Long geodesics in subgraphs of the cube*, Discrete Math. **326** (2014) 29–33.
[doi:10.1016/j.disc.2014.02.013](https://doi.org/10.1016/j.disc.2014.02.013)

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