ETERNAL m -SECURITY BONDAGE NUMBERS IN GRAPHS

HAMIDEH ARAM

Department of Mathematics
Gareziaeddin Center, Khoy Branch
Islamic Azad University, Khoy, Iran
e-mail: hamideh.aram@gmail.com

MARYAM ATAPOUR

Department of Mathematics
Faculty of Basic Sciences
University of Bonab, Bonab, I.R. Iran
e-mail: m.atapour@bonabu.ac.ir

AND

SEYED MAHMOUD SHEIKHOESLAMI

Department of Mathematics
Azərbaycan Şahid Mədanı University
Tabriz, I.R. Iran
e-mail: s.m.sheikholeslami@azaruniv.edu

Abstract

An eternal m -secure set of a graph $G = (V, E)$ is a set $S_0 \subseteq V$ that can defend against any sequence of single-vertex attacks by means of multiple guard shifts along the edges of G . The eternal m -security number $\sigma_m(G)$ is the minimum cardinality of an eternal m -secure set in G . The eternal m -security bondage number $b_{\sigma_m}(G)$ of a graph G is the minimum cardinality of a set of edges of G whose removal from G increases the eternal m -security number of G . In this paper, we study properties of the eternal m -security bondage number. In particular, we present some upper bounds on the eternal m -security bondage number in terms of eternal m -security number and edge connectivity number, and we show that the eternal m -security bondage number of trees is at most 2 and we classify all trees attaining this bound.

Keywords: eternal m -secure set, eternal m -security number, eternal m -security bondage number.

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