THE SUPER-CONNECTIVITY OF KNESER GRAPHS

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Abstract

A vertex cut of a connected graph G is a set of vertices whose deletion disconnects G . A connected graph G is super-connected if the deletion of every minimum vertex cut of G isolates a vertex. The super-connectivity is the size of the smallest vertex cut of G such that each resultant component does not have an isolated vertex. The Kneser graph $KG(n, k)$ is the graph whose vertices are the k -subsets of $\{1, 2, \dots, n\}$ and two vertices are adjacent if the k -subsets are disjoint. We use Baranyai's Theorem on the decompositions of complete hypergraphs to show that the Kneser graph $KG(n, 2)$ are super-connected when $n \geq 5$ and that their super-connectivity is $\binom{n}{2} - 6$ when $n \geq 6$.

Keywords: connectivity, super-connectivity, Kneser graphs.

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REFERENCES

- [1] C. Balbuena, X. Marcote and P. García-Vázquez, *On restricted connectivities of permutation graphs*, *Networks* **45** (2005) 113–118.
doi:10.1002/net.20056
- [2] Zs. Baranyai, *On the factorization of the complete uniform hypergraph*, *Colloq. Math. Soc. János Bolya* **10** (1975) 91–108.

- [3] F.T. Boesch, *Synthesis of reliable networks — a survey*, IEEE Trans. Reliability **35** (1986) 240–246.
doi:10.1109/TR.1986.4335424
- [4] F.T. Boesch and R. Tindell, *Circulants and their connectivities*, J. Graph Theory **8** (1984) 487–499.
doi:10.1002/jgt.3190080406
- [5] G. Boruzanlı Ekinci and J.B. Gauci, *On the reliability of generalized Petersen graphs*, Discrete Appl. Math., in press.
doi:10.1016/j.dam.2017.02.002
- [6] G. Boruzanlı Ekinci and A. Kırılgaç, *Super connectivity of Kronecker product of complete bipartite graphs and complete graphs*, Discrete Math. **339** (2016) 1950–1953.
doi:10.1016/j.disc.2015.10.036
- [7] B.-L. Chen and K.-W. Lih, *Hamiltonian uniform subset graphs*, J. Combin. Theory, Ser. B **42** (1987) 257–263.
doi:10.1016/0095-8956(87)90044-X
- [8] F. Harary, *Conditional connectivity*, Networks **13** (1983) 347–357.
doi:10.1002/net.3230130303
- [9] M.-C. Heydemann, *Cayley graphs and interconnection networks*, in: Graph Symmetry: Algebraic Methods and Applications, G. Hahn and G. Sabidussi (Ed(s)), (Springer, Dordrecht, 1997) 167–224.
doi:10.1007/978-94-015-8937-6_5
- [10] L.-H. Hsu and C.-K. Lin, Graph Theory and Interconnection Networks (CRC Press, 2008).
- [11] M. Kneser, *Aufgabe 360*, Jahresber. Dtsch. Math.-Ver. **58** (1955) 27.
- [12] M. Lü, C. Wu, G.-L. Chen and C. Lv, *On super connectivity of Cartesian product graphs*, Networks **52** (2008) 78–87.
doi:10.1002/net.20224
- [13] J.H. van Lint and R.M. Wilson, A Course in Combinatorics, 2nd edition (Cambridge University Press, Cambridge, 2001).
doi:10.1017/CBO9780511987045
- [14] M.E. Watkins, *Connectivity of transitive graphs*, J. Combin. Theory **8** (1970) 23–29.
doi:10.1016/S0021-9800(70)80005-9
- [15] W. Yang and J. Meng, *Extraconnectivity of hypercubes*, Appl. Math. Lett. **22** (2009) 887–891.
doi:10.1016/j.aml.2008.07.016
- [16] W. Yang and J. Meng, *Extraconnectivity of hypercubes (II)*, Australas. J. Combin. **47** (2010) 189–195.

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