FACIAL INCIDENCE COLORINGS OF EMBEDDED MULTIGRAPHS¹

MIRKO HORŇÁK, STANISLAV JENDROL'

AND

ROMAN SOTÁK

Institute of Mathematics
P.J. Šafárik University
Jesenná 5, 040 01 Košice, Slovakia

e-mail: {mirko.hornak, stanislav.jendrol, roman.sotak}@upjs.sk

Abstract

Let G be a cellular embedding of a multigraph in a 2-manifold. Two distinct edges $e_1, e_2 \in E(G)$ are facially adjacent if they are consecutive on a facial walk of a face $f \in F(G)$. An incidence of the multigraph G is a pair (v, e) , where $v \in V(G)$, $e \in E(G)$ and v is incident with e in G . Two distinct incidences (v_1, e_1) and (v_2, e_2) of G are facially adjacent if either $e_1 = e_2$ or e_1, e_2 are facially adjacent and either $v_1 = v_2$ or $v_1 \neq v_2$ and there is $i \in \{1, 2\}$ such that e_i is incident with both v_1, v_2 . A facial incidence coloring of G assigns a color to each incidence of G in such a way that facially adjacent incidences get distinct colors. In this note we show that any embedded multigraph has a facial incidence coloring with seven colors. This bound is improved to six for several wide families of plane graphs and to four for plane triangulations.

Keywords: embedded multigraph, incidence, facial incidence coloring.

2010 Mathematics Subject Classification: 05C15, 05C10.

REFERENCES

- [1] I. Algor and N. Alon, *The star arboricity of graphs*, Discrete Math. **75** (1989) 11–22.
doi:10.1016/0012-365X(89)90073-3

¹The research was supported by the Slovak grant VEGA 1/0368/16, by the grant APVV-15-0116, and by the Agency of the Ministry of Education, Science, Research and Sport of the Slovak Republic for the Structural Funds of EU under the project ITMS 26220120007.

- [2] K. Appel and W. Haken, Every Planar Map is Four Colorable (Contemporary Mathematics Vol. 98, American Mathematical Society, Providence, Rhode Island, 1989).
- [3] M. Axenovich, T. Ueckerdt and P. Weiner, *Splitting planar graphs of girth 6 into two linear forests with short paths*, J. Graph Theory **85** (2017) 601–618.
doi:10.1002/jgt.22093
- [4] M. Bonamy, B. Lvque and A. Pinlou, *2-distance coloring of sparse graphs*, J. Graph Theory **77** (2014) 190–218.
doi:10.1002/jgt.21782
- [5] R.A. Brualdi and J.J. Quinn Massey, *Incidence and strong edge colorings of graphs*, Discrete Math. **122** (1993) 51–58.
doi:10.1016/0012-365X(93)90286-3
- [6] I. Fabrici, S. Jendrol and M. Vrbjarov, *Unique-maximum edge-colouring of plane graphs with respect to faces*, Discrete Appl. Math. **185** (2015) 239–243.
doi:10.1016/j.dam.2014.12.002
- [7] W. Goddard, *Acyclic colorings of planar graphs*, Discrete Math. **91** (1991) 91–94.
doi:10.1016/0012-365X(91)90166-Y
- [8] B. Guiduli, *On incidence coloring and star arboricity of graphs*, Discrete Math. **163** (1997) 275–278.
doi:10.1016/0012-365X(95)00342-T
- [9] P. Hall, *On representatives of subsets*, J. Lond. Math. Soc. **10** (1935) 26–30.
doi:10.1112/jlms/s1-10.37.26
- [10] M. Hosseini Dolama, . Sopena and X. Zhu, *Incidence coloring of k -degenerated graphs*, Discrete Math. **283** (2004) 121–128.
doi:10.1016/j.disc.2004.01.015
- [11] M. Hosseini Dolama and . Sopena, *On the maximum average degree and the incidence chromatic number of a graph*, Discrete Math. Theor. Comput. Sci. **7** (2005) 203–216.
- [12] X. Li and J. Tu, *NP-completeness of 4-incidence colorability of semicubic graphs*, Discrete Math. **308** (2008) 1334–1340.
doi:10.1016/j.disc.2007.03.076
- [13] M. Maydanskyi, *The incidence coloring conjecture for graphs of maximum degree 3*, Discrete Math. **292** (2005) 131–141.
doi:10.1016/j.disc.2005.02.003
- [14] K.S. Poh, *On the linear vertex-arboricity of a planar graph*, J. Graph Theory **14** (1990) 73–75.
doi:10.1002/jgt.3190140108
- [15] N. Robertson, D. Sanders, P. Seymour and R. Thomas, *The four-colour theorem*, J. Combin. Theory Ser. B **70** (1997) 2–44.
doi:10.1006/jctb.1997.1750

- [16] W.C. Shiu, P.C.B. Lam and D.L. Chen, *On incidence coloring for some cubic graphs*, Discrete Math. **252** (2002) 259–266.
doi:10.1016/S0012-365X(01)00457-5
- [17] W.C. Shiu and P.K. Sun, *Invalid proofs on incidence coloring*, Discrete Math. **308** (2008) 6575–6580.
doi:10.1016/j.disc.2007.11.030
- [18] P.K. Sun, *Incidence coloring of regular graphs and complement graphs*, Taiwanese J. Math. **16** (2012) 2289–2295.
doi:10.11650/twjm/1500406852
- [19] W.F. Wang and K.W. Lih, *Labeling planar graphs with conditions on girth and distance two*, SIAM J. Discrete Math. **17** (2003) 264–275.
doi:10.1137/S0895480101390448
- [20] J. Wu, *Some results on the incidence coloring number of a graph*, Discrete Math. **309** (2009) 3866–3870.
doi:10.1016/j.disc.2008.10.027
- [21] D. Yang, *Fractional incidence coloring and star arboricity of graphs*, Ars Combin. **105** (2012) 213–224.

Received 12 December 2016

Revised 17 May 2017

Accepted 17 May 2017