THE SECOND NEIGHBOURHOOD FOR BIPARTITE TOURNAMENTS

RUIJUAN LI

School of Mathematical Sciences
Shanxi University
Taiyuan 030006, PR China

e-mail: ruijuanli@sxu.edu.cn

AND

BIN SHENG

College of Computer Science and Technology
Nanjing University of Aeronautics and Astronautics
Nanjing 211106, PR China

Department of Computer Science, Royal Holloway
University of London, TW20 0EX Egham, UK

e-mail: shengbinhello@nuaa.edu.cn

Abstract

Let $T(X \cup Y, A)$ be a bipartite tournament with partite sets X, Y and arc set A . For any vertex $x \in X \cup Y$, the second out-neighbourhood $N^{++}(x)$ of x is the set of all vertices with distance 2 from x . In this paper, we prove that T contains at least two vertices x such that $|N^{++}(x)| \geq |N^+(x)|$ unless T is in a special class $\mathcal{B}_1$ of bipartite tournaments; show that T contains at least a vertex x such that $|N^{++}(x)| \geq |N^-(x)|$ and characterize the class $\mathcal{B}_2$ of bipartite tournaments in which there exists exactly one vertex x with this property; and prove that if $|X| = |Y|$ or $|X| \geq 4|Y|$, then the bipartite tournament T contains a vertex x such that $|N^{++}(x)| + |N^+(x)| \geq 2|N^-(x)|$.

Keywords: second out-neighbourhood, out-neighbourhood, in-neighbourhood, bipartite tournament.

2010 Mathematics Subject Classification: 05C20, 05C12, 05C07.

REFERENCES

- [1] J. Bang-Jensen and G. Gutin, *Digraphs: Theory, Algorithms and Applications*, 2nd Ed. (Springer-Verlag, London, 2009).

- [2] J. Bang-Jensen and J. Huang, *Quasi-transitive digraphs*, J. Graph Theory **20** (1995) 141–161.
doi:10.1002/jgt.3190200205
- [3] G. Chen, J. Shen and R. Yuster, *Second neighborhood via first neighborhood in digraphs*, Ann. Combin. **7** (2003) 15–20.
doi:10.1007/s000260300001
- [4] Z. Cohn, A. Godbole, E. Wright Harkness and Y. Zhang, *The number of Seymour vertices in random tournaments and digraphs*, Graphs Combin. **32** (2016) 1805–1816.
doi:10.1007/s00373-015-1672-9
- [5] N. Dean and B.J. Latka, *Squaring the tournament—an open problem*, Congr. Numer. **109** (1995) 73–80.
- [6] D. Fidler and R. Yuster, *Remarks on the second neighborhood problem*, J. Graph Theory **55** (2007) 208–220.
doi:10.1002/jgt.v55:3
- [7] D.C. Fisher, *Squaring a tournament: a proof of Dean’s conjecture*, J. Graph Theory **23** (1996) 43–48.
doi:10.1002/(SICI)1097-0118(199609)23:1<43::AID-JGT4>3.0.CO;2-K
- [8] S. Ghazal, *Seymour’s second neighbourhood conjecture for tournaments missing a generalized star*, J. Graph Theory **71** (2012) 89–94.
doi:10.1002/jgt.20634
- [9] G. Gutin and R. Li, *Seymour’s second neighbourhood conjecture for quasi-transitive oriented graphs*.
arxiv.org/abs/1704.01389.
- [10] F. Havet and S. Thomassé, *Median orders of tournaments: a tool for the second neighborhood problem and Sumner’s conjecture*, J. Graph Theory **35** (2000) 244–256.
doi:10.1002/1097-0118(200012)35:4<244::AID-JGT2>3.0.CO;2-H
- [11] Y. Kaneko and S.C. Locke, *The minimum degree approach for Paul Seymour’s distance 2 conjecture*, Congr. Numer. **148** (2001) 201–206.
- [12] J.W. Moon, *Solution to problem 463*, Math. Mag. **35** (1962) 189.
doi:10.2307/2688555
- [13] B. Sullivan, *A summary of results and problems related to the Caccetta-Häggkvist conjecture*.
arxiv.org/abs/math/0605646.
- [14] R. Li and B. Sheng, *The second neighbourhood for quasi-transitive oriented graphs*, submitted.

Received 12 June 2017
Revised 13 October 2017
Accepted 13 October 2017