GENERALIZED RAINBOW CONNECTION OF GRAPHS AND THEIR COMPLEMENTS

XUELIANG LI¹, COLTON MAGNANT²

MEIQIN WEI¹ AND XIAOYU ZHU¹

¹*Center for Combinatorics and LPMC
Nankai University, Tianjin 300071, China*

²*Department of Mathematical Sciences
Georgia Southern University
Statesboro, GA 30460-8093, USA*

e-mail: lxl@nankai.edu.cn
cmagnant@georgiasouthern.edu
weimeiqin8912@163.com
zhuxy@mail.nankai.edu.cn

Abstract

Let G be an edge-colored connected graph. A path P in G is called ℓ -rainbow if each subpath of length at most $\ell + 1$ is rainbow. The graph G is called (k, ℓ) -rainbow connected if there is an edge-coloring such that every pair of distinct vertices of G is connected by k pairwise internally vertex-disjoint ℓ -rainbow paths in G . The minimum number of colors needed to make G (k, ℓ) -rainbow connected is called the (k, ℓ) -rainbow connection number of G and denoted by $rc_{k,\ell}(G)$. In this paper, we first focus on the $(1, 2)$ -rainbow connection number of G depending on some constraints of $\overline{G}$. Then, we characterize the graphs of order n with $(1, 2)$ -rainbow connection number $n - 1$ or $n - 2$. Using this result, we investigate the Nordhaus-Gaddum-Type problem of $(1, 2)$ -rainbow connection number and prove that $rc_{1,2}(G) + rc_{1,2}(\overline{G}) \leq n + 2$ for connected graphs G and $\overline{G}$. The equality holds if and only if G or $\overline{G}$ is isomorphic to a double star.

Keywords: ℓ -rainbow path, (k, ℓ) -rainbow connected, (k, ℓ) -rainbow connection number.

2010 Mathematics Subject Classification: 04C15, 05C40.

REFERENCES

- [1] E. Andrews, E. Laforge, C. Lunduanhom and P. Zhang, *On proper-path colorings in graphs*, J. Combin. Math. Combin. Comput. **97** (2016) 189–207.
- [2] V. Borozan, S. Fujita, A. Gerek, C. Magnant, Y. Manoussakis, L. Montero and Zs. Tuza, *Proper connection of graphs*, Discrete Math. **312** (2012) 2550–2560.
doi:10.1016/j.disc.2011.09.003
- [3] J.A. Bondy and U.S.R. Murty, Graph Theory (GTM 244, Springer-Verlag, New York, 2008).
- [4] G. Chartrand, S. Devereaux and P. Zhang, *Color-connected graphs and information-transfer paths*, Ars Combin., to appear.
- [5] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *Rainbow connection in graphs*, Math. Bohem. **133** (2008) 85–98.
- [6] S. Devereaux, G.L. Johns and P. Zhang, *Color connection in graphs intermediate to proper and rainbow connection*, J. Combin. Math. Combin. Comput., to appear.
- [7] S. Devereaux and P. Zhang, *k-rainbow colorings in graphs*, manuscript.
- [8] J.L. Fouquet and J.L. Jolivet, *Strong edge-coloring of graphs and applications to multi-k-gons*, Ars Combin. **16A** (1983) 141–150.
- [9] M. Krivelevich and R. Yuster, *The rainbow connection of a graph is (at most) reciprocal to its minimum degree*, J. Graph Theory **63** (2010) 185–191.
doi:10.1002/jgt.20418
- [10] X. Li and C. Magnant, *Properly colored notions of connectivity—a dynamic survey*, Theory Appl. Graphs **0(1)** (2015) Article 2.
doi:10.20429/tag.2015.000102
- [11] X. Li, C. Magnant, M. Wei and X. Zhu, *Distance proper connection of graphs* (2016) arXiv:1606.06547 [math.CO]
- [12] X. Li and Y. Shi, *Rainbow connection in 3-connected graphs*, Graphs Combin. **29** (2013) 1471–1475.
doi:10.1007/s00373-012-1204-9
- [13] X. Li, Y. Shi and Y. Sun, *Rainbow connections of graphs: A survey*, Graphs Combin. **29** (2013) 1–38.
doi:10.1007/s00373-012-1243-2
- [14] X. Li and Y. Sun, *Rainbow Connections of Graphs* (Springer Briefs in Math., Springer, New York, 2012).

Received 26 July 2016
 Revised 26 November 2016
 Accepted 29 November 2016