A SHARP LOWER BOUND FOR THE GENERALIZED 3-EDGE-CONNECTIVITY OF STRONG PRODUCT GRAPHS

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Abstract

The generalized k -connectivity $\kappa_k(G)$ of a graph G , mentioned by Hager in 1985, is a natural generalization of the path-version of the classical connectivity. As a natural counterpart of this concept, Li *et al.* in 2011 introduced the concept of generalized k -edge-connectivity which is defined as $\lambda_k(G) = \min\{\lambda_G(S) \mid S \subseteq V(G) \text{ and } |S| = k\}$, where $\lambda_G(S)$ denote the maximum number ℓ of pairwise edge-disjoint trees $T_1, T_2, \dots, T_\ell$ in G such that $S \subseteq V(T_i)$ for $1 \leq i \leq \ell$. In this paper we get a sharp lower bound for the generalized 3-edge-connectivity of the strong product of any two connected graphs.

Keywords: generalized connectivity, generalized edge-connectivity, strong product.

2010 Mathematics Subject Classification: 05C05, 05C40, 05C76.

REFERENCES

- [1] J.A. Bondy and U.S.R. Murty, Graph Theory, GTM 244 (Springer, Berlin, 2008).
- [2] B. Brešar and S. Špacapan, *Edge-connectivity of strong products of graphs*, Discuss. Math. Graph Theory **27** (2007) 333–343.
doi:10.7151/dmgt.1365
- [3] G. Chartrand, S.F. Kappor, L. Lesniak and D.R. Lick, *Generalized connectivity in graphs*, Bull. Bombay Math. Colloq. **2** (1984) 1–6.
- [4] W.S. Chiue and B.S. Shieh, *On connectivity of the Cartesian product of two graphs*, Appl. Math. Comput. **102** (1999) 129–137.
doi:10.1016/S0096-3003(98)10041-3

- [5] D.P. Day, O.R. Oellermann and H.C. Swart, *The ℓ -connectivity function of trees and complete multipartite graphs*, J. Combin. Math. Combin. Comput. **10** (1991) 183–192.
- [6] D. Du and X. Hu, *Steiner Tree Problems in Computer Communication Networks* (World Scientific, 2008).
doi:10.1142/6729
- [7] M. Grötschel, *The Steiner tree packing problem in VLSI design*, Math. Program. **78** (1997) 265–281.
doi:10.1007/BF02614374
- [8] M. Grötschel, A. Martin and R. Weismantel, *Packing Steiner trees: A cutting plane algorithm and computational results*, Math. Program. **72** (1996) 125–145.
doi:10.1007/BF02592086
- [9] M. Hager, *Pendant tree-connectivity*, J. Combin. Theory Ser. B **38** (1985) 179–189.
doi:10.1016/0095-8956(85)90083-8
- [10] R. Hammack, W. Imrich and S. Klavžar, *Handbook of Product Graphs*, Second Edition (CRC Press, 2011).
- [11] W. Imrich and S. Klavžar, *Product Graphs—Structure and Recognition* (Wiley, New York, 2000).
- [12] W. Imrich, S. Klavžar and D.F. Rall, *Topics in Graph Theory: Graphs and Their Cartesian Product* (A K Peters, 2008).
- [13] S. Klavžar and S. Špacapan, *On the edge-connectivity of Cartesian product graphs*, Asian-Eur. J. Math. **1** (2008) 93–98.
- [14] H. Li, X. Li, Y. Mao and Y. Sun, *Note on the generalized connectivity*, Ars Combin. **114** (2014) 193–202.
- [15] H. Li, X. Li and Y. Sun, *The generalized 3-connectivity of Cartesian product graphs*, Discrete Math. Theor. Comput. Sci. **14** (2012) 43–54.
- [16] X. Li and Y. Mao, *A survey on the generalized connectivity of graphs*.
arXiv:1207.1838[math.CO]
- [17] X. Li and Y. Mao, *On extremal graphs with at most ℓ internally disjoint Steiner trees connecting any $n - 1$ vertices*, Graphs Combin. **31** (2015) 1–29.
doi:10.1007/s00373-014-1465-6
- [18] X. Li and Y. Mao, *The generalized 3-connectivity of lexicographic product graphs*, Discrete Math. Theor. Comput. Sci. **16** (2014) 339–354.
doi:10.1007/978-3-319-12691-3_31
- [19] X. Li and Y. Mao, *Nordhaus-Gaddum-type results for the generalized edge-connectivity of graphs*, Discrete Appl. Math. **185** (2015) 102–112.
doi:10.1016/j.dam.2014.12.009
- [20] X. Li, Y. Mao and Y. Sun, *On the generalized (edge-)connectivity of graphs*, Australas. J. Combin. **58** (2014) 304–319.

- [21] X. Li, Y. Mao and L. Wang, *Graphs with large generalized 3-edge-connectivity*, arXiv:1201.3699v1 [math.CO]
- [22] X. Li, J. Yue, and Y. Zhao, *The generalized 3-edge-connectivity of lexicographic product graphs*, Proc. COCOA 2014, Lecture Notes in Comput. Sci. **8881** (2014) 412–425.
doi:10.1007/978-3-319-12691-3_31
- [23] B. Liouville, *Sur la connectivité des produits de graphes*, C.R. Acad. Sci. Paris Sér. A–B **286** (1978) A363–A365.
- [24] O.R. Oellermann, *On the ℓ -connectivity of a graph*, Graphs Combin. **3** (1987) 285–299.
doi:10.1007/BF01788551
- [25] O.R. Oellermann, *A note on the ℓ -connectivity function of a graph*, Congr. Numer. **60** (1987) 181–188.
- [26] G. Sabidussi, *Graphs with given group and given graph theoretical properties*, Canadian J. Math. **9** (1957) 515–525.
doi:0.4153/CJM-1957-060-7
- [27] N.A. Sherwani, *Algorithms for VLSI Physical Design Automation*, 3rd Edition (Kluwer Acad. Pub., London, 1999).
- [28] Y. Sun, *Generalized 3-(edge)-connectivity for undirected double-loop networks*, J. Discrete Math. Sci. Cryptogr. **17** (2014) 19–28.
doi:10.1080/09720529.2013.867672
- [29] Y. Sun, *Generalized 3-edge-connectivity of Cartesian product graphs*, Czechoslovak Math. J. **65** (2015) 107–117.
doi:10.1007/s10587-015-0162-9
- [30] Y. Sun, *Generalized 3-connectivity and 3-edge-connectivity for the Cartesian products of some graph classes*, J. Combin. Math. Combin. Comput. **94** (2015) 215–225.
- [31] Y. Sun, *Maximum generalized local connectivities of cubic Cayley graphs on Abelian groups*, J. Combin. Math. Combin. Comput. **94** (2015) 227–236.
- [32] Y. Sun, *Sharp upper bounds for generalized edge-connectivity of product graphs*, Discuss. Math. Graph Theory **36** (2016) 833–843.
doi:10.7151/dmgt.1924
- [33] Y. Sun, *On the maximum and minimum sizes of a graph with given k -connectivity*, Discuss. Math. Graph Theory **37** (2017) 623–632.
doi:10.7151/dmgt.1941
- [34] Y. Sun and X. Li, *On the difference of two generalized connectivities of a graph*, J. Comb. Optim. **33** (2017) 283–291.
doi:10.1007/s10878-015-9956-9.
- [35] Y. Sun and S. Zhou, *Tree connectivities of Cayley graphs on Abelian groups with small degrees*, Bull. Malays. Math. Sci. Soc. **39** (2016) 1673–1685.
doi:10.1007/s40840-015-0147-8.

- [36] S. Špacapan, *Connectivity of Cartesian products of graphs*, Appl. Math. Lett. **21** (2008) 682–685.
doi:10.1016/j.aml.2007.06.010
- [37] S. Špacapan, *Connectivity of strong products of graphs*, Graphs Combin. **26** (2010) 457–467.
doi:10.1007/s00373-010-0919-8
- [38] J. Xu and C. Yang, *Connectivity of Cartesian product graphs*, Discrete Math. **306** (2006) 159–165.
doi:10.1016/j.disc.2005.11.010

Received 13 February 2016

Revised 17 August 2016

Accepted 17 August 2016