ON THE SPECTRAL CHARACTERIZATIONS OF GRAPHS

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Abstract

Several matrices can be associated to a graph, such as the adjacency matrix or the Laplacian matrix. The spectrum of these matrices gives some informations about the structure of the graph and the question “Which graphs are determined by their spectrum?” is still a difficult problem in spectral graph theory. Let $\mathcal{W}_p^{2q}$ be the set of graphs obtained from C_p by attaching two pendant edges to each of q ($q \leq p$) vertices on C_p , whereas $\mathcal{V}_p^{2q}$ the subset of $\mathcal{W}_p^{2q}$ with odd p and its q vertices of degree 4 being nonadjacent to each other. In this paper, we show that each graph in $\mathcal{W}_p^{2q}$, p even and its q vertices of degree 4 being consecutive, is determined by its Laplacian spectrum. As well we show that if G is a graph without isolated vertices and adjacency cospectral with the graph in $\mathcal{V}_p^{p-1} = \{H\}$, then $G \cong H$.

Keywords: Laplacian spectrum, adjacency spectrum, cospectral graphs, spectral characterization.

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