A DEGREE CONDITION IMPLYING ORE-TYPE CONDITION FOR EVEN $[2, b]$ -FACTORS IN GRAPHS

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Abstract

For a graph G and even integers $b \geq a \geq 2$, a spanning subgraph F of G such that $a \leq \deg_F(x) \leq b$ and $\deg_F(x)$ is even for all $x \in V(F)$ is called an even $[a, b]$ -factor of G . In this paper, we show that a 2-edge-connected graph G of order n has an even $[2, b]$ -factor if $\max\{\deg_G(x), \deg_G(y)\} \geq \max\{\frac{2n}{2+b}, 3\}$ for any nonadjacent vertices x and y of G . Moreover, we show that for $b \geq 3a$ and $a > 2$, there exists an infinite family of 2-edge-connected graphs G of order n with $\delta(G) \geq a$ such that G satisfies the condition $\deg_G(x) + \deg_G(y) > \frac{2an}{a+b}$ for any nonadjacent vertices x and y of G , but has no even $[a, b]$ -factors. In particular, the infinite family of graphs gives a counterexample to the conjecture of Matsuda on the existence of an even $[a, b]$ -factor.

Keywords: $[a, b]$ -factor, even factor, 2-edge-connected, minimum degree.

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