KALEIDOSCOPIC COLORINGS OF GRAPHS

GARY CHARTRAND, SEAN ENGLISH AND PING ZHANG

Department of Mathematics
Western Michigan University
Kalamazoo, MI 49008, USA

e-mail: gary.chartrand@wmich.edu

Abstract

For an r -regular graph G , let $c : E(G) \rightarrow [k] = \{1, 2, \dots, k\}$, $k \geq 3$, be an edge coloring of G , where every vertex of G is incident with at least one edge of each color. For a vertex v of G , the multiset-color $c_m(v)$ of v is defined as the ordered k -tuple $(a_1, a_2, \dots, a_k)$ or $a_1 a_2 \cdots a_k$, where a_i ($1 \leq i \leq k$) is the number of edges in G colored i that are incident with v . The edge coloring c is called k -kaleidoscopic if $c_m(u) \neq c_m(v)$ for every two distinct vertices u and v of G . A regular graph G is called a k -kaleidoscope if G has a k -kaleidoscopic coloring. It is shown that for each integer $k \geq 3$, the complete graph K_{k+3} is a k -kaleidoscope and the complete graph K_n is a 3-kaleidoscope for each integer $n \geq 6$. The largest order of an r -regular 3-kaleidoscope is $\binom{r-1}{2}$. It is shown that for each integer $r \geq 5$ such that $r \not\equiv 3 \pmod{4}$, there exists an r -regular 3-kaleidoscope of order $\binom{r-1}{2}$.

Keywords: edge coloring, vertex coloring, kaleidoscopic coloring, kaleidoscope.

2010 Mathematics Subject Classification: 05C15, 05C75.

REFERENCES

- [1] M. Aigner, E. Triesch and Zs. Tuza, *Irregular assignments and vertex-distinguishing edge-colorings of graphs*, Combinatorics' 90 Elsevier Science Pub., New York (1992) 1–9.
- [2] A C. Burris and R.H. Schelp, *Vertex-distinguishing proper edge colorings*, J. Graph Theory **26** (1997) 73–82.
doi:10.1002/(SICI)1097-0118(199710)26:2<73::AID-JGT2>3.0.CO;2-C
- [3] J. Černý, M. Horňák and R. Soták, *Observability of a graph*, Math. Slovaca **46** (1996) 21–31.

- [4] G. Chartrand, S. English and P. Zhang, *Binomial colorings of graphs*, Bull. Inst. Combin. Appl. **76** (2016) 69–84.
- [5] G. Chartrand and P. Zhang, *Chromatic Graph Theory* (Chapman & Hall/CRC Press, Boca Raton, FL, 2009).
- [6] O. Favaron, H. Li and R. H. Schelp, *Strong edge colorings of graphs*, Discrete Math. **159** (1996) 103–109.
doi:10.1016/0012-365X(95)00102-3
- [7] P.G. Tait, *Remarks on the colouring of maps*, Proc. Royal Soc. Edinburgh **10** (1880) 501–503, 729.
doi:10.1017/S0370164600044643

Received 11 September 2015

Revised 18 April 2016

Accepted 13 June 2016