CRITICALITY OF SWITCHING CLASSES OF REVERSIBLE 2-STRUCTURES LABELED BY AN ABELIAN GROUP

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Abstract

Let V be a finite vertex set and let $(\mathbb{A}, +)$ be a finite abelian group. An $\mathbb{A}$ -labeled and reversible 2-structure defined on V is a function $g : (V \times V) \setminus \{(v, v) : v \in V\} \rightarrow \mathbb{A}$ such that for distinct $u, v \in V$, $g(u, v) = -g(v, u)$. The set of $\mathbb{A}$ -labeled and reversible 2-structures defined on V is denoted by $\mathcal{L}(V, \mathbb{A})$. Given $g \in \mathcal{L}(V, \mathbb{A})$, a subset X of V is a *clan* of g if for any $x, y \in X$ and $v \in V \setminus X$, $g(x, v) = g(y, v)$. For example, $\emptyset$, V and $\{v\}$ (for $v \in V$) are clans of g , called *trivial*. An element g of $\mathcal{L}(V, \mathbb{A})$ is *primitive* if $|V| \geq 3$ and all the clans of g are trivial.

The set of the functions from V to $\mathbb{A}$ is denoted by $\mathcal{S}(V, \mathbb{A})$. Given $g \in \mathcal{L}(V, \mathbb{A})$, with each $s \in \mathcal{S}(V, \mathbb{A})$ is associated the *switch* g^s of g by s defined as follows: given distinct $x, y \in V$, $g^s(x, y) = s(x) + g(x, y) - s(y)$. The *switching class* of g is $\{g^s : s \in \mathcal{S}(V, \mathbb{A})\}$. Given a switching class $\mathfrak{S} \subseteq \mathcal{L}(V, \mathbb{A})$ and $X \subseteq V$, $\{g|_{(X \times X) \setminus \{(x, x) : x \in X\}} : g \in \mathfrak{S}\}$ is a switching class, denoted by $\mathfrak{S}[X]$.

Given a switching class $\mathfrak{S} \subseteq \mathcal{L}(V, \mathbb{A})$, a subset X of V is a *clan* of $\mathfrak{S}$ if X is a clan of some $g \in \mathfrak{S}$. For instance, every $X \subseteq V$ such that $\min(|X|, |V \setminus X|) \leq 1$ is a clan of $\mathfrak{S}$, called *trivial*. A switching class $\mathfrak{S} \subseteq \mathcal{L}(V, \mathbb{A})$ is *primitive* if $|V| \geq 4$ and all the clans of $\mathfrak{S}$ are trivial. Given a primitive switching class $\mathfrak{S} \subseteq \mathcal{L}(V, \mathbb{A})$, $\mathfrak{S}$ is *critical* if for each $v \in V$, $\mathfrak{S} - v$ is not primitive. First, we translate the main results on the primitivity of $\mathbb{A}$ -labeled and reversible 2-structures in terms of switching classes. For instance, we prove the following. For a primitive switching class $\mathfrak{S} \subseteq \mathcal{L}(V, \mathbb{A})$ such that $|V| \geq 8$, there exist $u, v \in V$ such that $u \neq v$ and $\mathfrak{S}[V \setminus \{u, v\}]$ is primitive. Second, we characterize the critical switching classes by using some of the critical digraphs described in [Y. Boudabous and P. Ille, *Indecomposability graph and critical vertices of an indecomposable graph*, Discrete Math. **309** (2009) 2839–2846].

Keywords: labeled and reversible 2-structure, switching class, clan, primitivity, criticality.

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REFERENCES

- [1] P. Bonizzoni, *Primitive 2-structures with the $(n - 2)$ -property*, Theoret. Comput. Sci. **132** (1994) 151–178.
doi:10.1016/0304-3975(94)90231-3
- [2] Y. Boudabous and P. Ille, *Indecomposability graph and critical vertices of an indecomposable graph*, Discrete Math. **309** (2009) 2839–2846.
doi:10.1016/j.disc.2008.07.015
- [3] Y. Cheng and A.L. Wells, *Switching classes of directed graphs*, J. Combin. Theory Ser. B **40** (1986) 169–186.
doi:10.1016/0095-8956(86)90075-4
- [4] A. Cournier and P. Ille, *Minimal indecomposable graph*, Discrete Math. **183** (1998) 61–80.
doi:10.1016/S0012-365X(97)00077-0
- [5] A. Ehrenfeucht, T. Harju and G. Rozenberg, *The Theory of 2-Structures, A Framework for Decomposition and Transformation of Graphs* (World Scientific, 1999).
doi:10.1142/4197
- [6] A. Ehrenfeucht and G. Rozenberg, *Theory of 2-structures, Part I: clans, basic subclasses, and morphisms*, Theoret. Comput. Sci. **70** (1990) 277–303.
doi:10.1016/0304-3975(90)90129-6
- [7] A. Ehrenfeucht and G. Rozenberg, *Primitivity is hereditary for 2-structures*, Theoret. Comput. Sci. **70** (1990) 343–358.
- [8] P. Ille, *Recognition problem in reconstruction for decomposable relations*, in: Finite and Infinite Combinatorics in Sets and Logic, B. Sands, N. Sauer and R. Woodrow (Ed(s)), (Kluwer Academic Publishers, 1993) 189–198.
doi:10.1007/978-94-011-2080-7_13

- [9] P. Ille, *Indecomposable graphs*, Discrete Math. **173** (1997) 71–78.
doi:10.1016/S0012-365X(96)00097-0
- [10] P. Ille and R. Woodrow, *Decomposition tree of a lexicographic product of binary structures*, Discrete Math. **311** (2011) 2346–2358.
doi:10.1016/j.disc.2011.05.037
- [11] J.H. Schmerl and W.T. Trotter, *Critically indecomposable partially ordered sets, graphs, tournaments and other binary relational structures*, Discrete Math. **113** (1993) 191–205.
doi:10.1016/0012-365X(93)90516-V
- [12] J.J. Seidel, *Strongly regular graphs of L2 type and of triangular type*, in: Proc. Kon. Nederl. Akad. Wetensch. Ser. A (Indag. Math. **29**, 1967) 188–196.
doi:10.1016/S1385-7258(67)50031-8

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