THE THICKNESS OF AMALGAMATIONS AND CARTESIAN PRODUCT OF GRAPHS

YAN YANG

Department of Mathematics
Tianjin University
300072, Tianjin, China

e-mail: yanyang@tju.edu.cn

AND

YICHAO CHEN

College of Mathematics and Econometrics
Hunan University
410082 Changsha, China

e-mail: ycchen@hnu.edu.cn

Abstract

The thickness of a graph is the minimum number of planar spanning subgraphs into which the graph can be decomposed. It is a measurement of the closeness to the planarity of a graph, and it also has important applications to VLSI design, but it has been known for only few graphs. We obtain the thickness of vertex-amalgamation and bar-amalgamation of graphs, the lower and upper bounds for the thickness of edge-amalgamation and 2-vertex-amalgamation of graphs, respectively. We also study the thickness of Cartesian product of graphs, and by using operations on graphs, we derive the thickness of the Cartesian product $K_n \square P_m$ for most values of m and n .

Keywords: thickness, amalgamation, Cartesian product, genus.

2010 Mathematics Subject Classification: 05C10.

REFERENCES

- [1] A. Aggarwal, M. Klawe and P. Shor, *Multilayer grid embeddings for VLSI*, *Algorithmica* **6** (1991) 129–151.
doi:10.1007/BF01759038

- [2] S. Alpert, *The genera of amalgamations of graph*, Trans. Amer. Math. Soc. **178** (1973) 1–39.
doi:10.1090/S0002-9947-1973-0371698-X
- [3] V.B. Alekseev and V.S. Gončakov, *The thickness of an arbitrary complete graph*, Math. Sbornik. **30** (1976) 187–202.
doi:10.1070/SM1976v030n02ABEH002267
- [4] K. Asano, *On the genus and thickness of graphs*, J. Combin. Theory Ser. B **43** (1987) 287–292.
doi:10.1016/0095-8956(87)90004-9
- [5] J. Battle, F. Harary, Y. Kodama and J.W.T. Youngs, *Additivity of the genus of a graph*, Bull. Amer. Math. Soc. **68** (1962) 565–568.
doi:10.1090/S0002-9904-1962-10847-7
- [6] L.W. Beineke and F. Harary, *The thickness of the complete graph*, Canad. J. Math. **17** (1965) 850–859.
doi:10.4153/CJM-1965-084-2
- [7] L.W. Beineke, F. Harary and J.W. Moon, *On the thickness of the complete bipartite graph*, Math. Proc. Cambridge Philos. Soc. **60** (1964) 1–5.
doi:10.1017/S0305004100037385
- [8] M. Behzad and S.E. Mahmoodian, *On topological invariants of the product of graphs*, Canad. Math. Bull. **12** (1969) 157–166.
doi:10.4153/CMB-1969-015-9
- [9] J.A. Bondy and U.S.R. Murty, *Graph Theory* (Springer, 2008).
- [10] J.E. Chen, S.P. Kanchi and A. Kanevsky, *A note on approximating graph genus*, Inform. Process Lett. **61** (1997) 317–322.
doi:10.1016/S0020-0190(97)00203-2
- [11] R.J. Cimikowski, *On heuristics for determining the thickness of a graph*, Inform. Sci. **85** (1995) 87–98.
doi:10.1016/0020-0255(95)00011-D
- [12] A.M. Dean, J.P. Hutchinson and E.R. Scheinerman, *On the thickness and arboricity of a graph*, J. Combin. Theory Ser. B **52** (1991) 147–151.
doi:10.1016/0095-8956(91)90100-X
- [13] R. Decker, H. Glover and J.P. Huneke, *The genus of the 2-amalgamations of graphs*, J. Graph Theory **5** (1981) 95–102.
doi:10.1002/jgt.3190050107
- [14] J.H. Halton, *On the thickness of graphs of given degree*, Inform. Sci. **54** (1991) 219–238.
doi:10.1016/0020-0255(91)90052-V
- [15] A.M. Hobbs, *A survey of thickness*, in: Recent Progress in Combinatorics (Proc. 3rd Waterloo Conf. on Combinatorics, 1968), W.T. Tutte (Ed(s)), (New York, Academic Press, 1969) 255–264.

- [16] M. Kleinert, *Die Dicke des n -dimensionalen Würfel-Graphen*, J. Combin. Theory **3** (1967) 10–15.
doi:10.1016/S0021-9800(67)80010-3
- [17] A. Mansfield, *Determining the thickness of graphs is NP-hard*, Math. Proc. Cambridge Philos. Soc. **93** (1983) 9–23.
doi:10.1017/S030500410006028X
- [18] E. Mäkinen and T. Poranen, *An annotated bibliography on the thickness, outerthickness, and arboricity of a graph*, Missouri J. Math. Sci. **24** (2012) 76–87.
- [19] P. Mutzel, T. Odenthal and M. Scharbrodt, *The thickness of graphs: a survey*, Graphs Combin. **14** (1998) 59–73.
doi:10.1007/PL00007219
- [20] T. Poranen, *A simulated annealing algorithm for determining the thickness of a graph*, Inform. Sci. **172** (2005) 155–172.
doi:10.1016/j.ins.2004.02.029
- [21] W.T. Tutte, *The thickness of a graph*, Indag. Math. **25** (1963) 567–577.
doi:10.1016/S1385-7258(63)50055-9
- [22] J.M. Vasak, *The thickness of the complete graph*, Notices Amer. Math. Soc. **23** (1976) A-479.

Received 29 June 2015

Revised 18 May 2016

Accepted 20 May 2016