C_7 -DECOMPOSITIONS OF THE TENSOR PRODUCT OF COMPLETE GRAPHS

R.S. MANIKANDAN

*Department of Mathematics
Bharathidasan University Constituent College
Lalgudi-621 601, India*

e-mail: manirs2004@yahoo.co.in

AND

P. PAULRAJA

*Department of Mathematics
Kalasalingam University
Krishnankoil-626 126, India*

e-mail: ppraja56@gmail.com

Abstract

In this paper we consider a decomposition of $K_m \times K_n$, where $\times$ denotes the tensor product of graphs, into cycles of length seven. We prove that for $m, n \geq 3$, cycles of length seven decompose the graph $K_m \times K_n$ if and only if (1) either m or n is odd and (2) $14 \mid m(m-1)n(n-1)$. The results of this paper together with the results of [C_p -Decompositions of some regular graphs, Discrete Math. **306** (2006) 429–451] and [C_5 -Decompositions of the tensor product of complete graphs, Australasian J. Combinatorics **37** (2007) 285–293], give necessary and sufficient conditions for the existence of a p -cycle decomposition, where $p \geq 5$ is a prime number, of the graph $K_m \times K_n$.

Keywords: cycle decomposition, tensor product.

2010 Mathematics Subject Classification: 05C70.

REFERENCES

- [1] B. Alspach and H. Gavlas, *Cycle decompositions of K_n and $K_n - I$* , J. Combin. Theory Ser. B **81** (2001) 77–99.
doi:10.1006/jctb.2000.1996

- [2] B. Alspach, P.J. Schellenberg, D.R. Stinson and D. Wagner, *The Oberwolfach problem and factors of uniform odd length cycles*, J. Combin. Theory Ser. A **52** (1989) 20–43.
doi:10.1016/0097-3165(89)90059-9
- [3] E.J. Billington, *Decomposing complete tripartite graphs into cycles of length 3 and 4*, Discrete Math. **197/198** (1999) 123–135.
doi:10.1016/S0012-365X(98)00227-1
- [4] E.J. Billington, D.G. Hoffman and B.M. Maenhaut, *Group divisible pentagon systems*, Util. Math. **55** (1999) 211–219.
- [5] N.J. Cavenagh and E.J. Billington, *Decompositions of complete multipartite graphs into cycles of even length*, Graphs Combin. **16** (2000) 49–65.
doi:10.1007/s003730050003
- [6] N.J. Cavenagh and E.J. Billington, *On decomposing complete tripartite graphs into 5-cycles*, Australas. J. Combin. **22** (2000) 41–62.
- [7] C.C. Lindner and C.A. Rodger, *Design Theory* (CRC Press New York, 1997).
- [8] J. Liu, *A Generalization of the Oberwolfach problem and C_t -factorizations of complete equipartite graphs*, J. Combin. Designs **8** (2000) 42–49.
doi:10.1002/(SICI)1520-6610(2000)8:1<42::AID-JCD6>3.0.CO;2-R
- [9] E.S. Mahmoodian and M. Mirzakhani, *Decomposition of complete tripartite graphs into 5-cycles*, in: Combinatorics and Advances, C.J. Colbourn and E.S. Mahmoodian (Ed(s)), (Kluwer Academic Publ., 1995) 235–241.
- [10] R.S. Manikandan and P. Paulraja, *C_p -decompositions of some regular graphs*, Discrete Math. **306** (2006) 429–451.
doi:10.1016/j.disc.2005.08.006
- [11] R.S. Manikandan and P. Paulraja, *C_5 -decompositions of the tensor product of complete graphs*, Australas. J. Combin. **37** (2007) 285–293.
- [12] A. Muthusamy and P. Paulraja, *Factorizations of product graphs into cycles of uniform length*, Graphs Combin. **11** (1995) 69–90.
doi:10.1007/BF01787423
- [13] M. Šajna, *Cycle decompositions III: Complete graphs and fixed length cycles*, J. Combin. Designs **10** (2002) 27–78.
doi:10.1002/jcd.1027
- [14] B.R. Smith, *Decomposing complete equipartite graphs into cycles of length $2p$* , J. Combin. Designs **16** (2008) 244–252.
doi:10.1002/jcd.20173
- [15] B.R. Smith, *Complete equipartite $3p$ -cycle systems*, Australas. J. Combin. **45** (2009) 125–138.
- [16] B.R. Smith, *Decomposing complete equipartite graphs into odd square-length cycles: number of parts odd*, J. Combin. Designs **18** (2010) 401–414.
doi:10.1002/jcd.20268

- [17] B.R. Smith, *Cycle decompositions of λ -fold complete equipartite graphs*, Australas. J. Combin. **47** (2010) 145–156.

Received 14 December 2015

Revised 5 May 2016

Accepted 5 May 2016