SOME RESULTS ON 4-TRANSITIVE DIGRAPHS

PATRICIO RICARDO GARCÍA-VÁZQUEZ

AND

CÉSAR HERNÁNDEZ-CRUZ

Facultad de Ciencias
Universidad Nacional Autónoma de México
Ciudad Universitaria, México, D.F.

e-mail: patricio@matem.unam.mx
chc@ciencias.unam.mx

Abstract

Let D be a digraph with set of vertices V and set of arcs A . We say that D is k -transitive if for every pair of vertices $u, v \in V$, the existence of a uv -path of length k in D implies that $(u, v) \in A$. A 2-transitive digraph is a transitive digraph in the usual sense.

A subset N of V is k -independent if for every pair of vertices $u, v \in N$, we have $d(u, v), d(v, u) \geq k$; it is l -absorbent if for every $u \in V \setminus N$ there exists $v \in N$ such that $d(u, v) \leq l$. A k -kernel of D is a k -independent and $(k - 1)$ -absorbent subset of V . The problem of determining whether a digraph has a k -kernel is known to be $\mathcal{NP}$ -complete for every $k \geq 2$.

In this work, we characterize 4-transitive digraphs having a 3-kernel and also 4-transitive digraphs having a 2-kernel. Using the latter result, a proof of the Laborde-Payan-Xuong conjecture for 4-transitive digraphs is given. This conjecture establishes that for every digraph D , an independent set can be found such that it intersects every longest path in D . Also, Seymour's Second Neighborhood Conjecture is verified for 4-transitive digraphs and further problems are proposed.

Keywords: 4-transitive digraph, k -transitive digraph, 3-kernel, k -kernel, Laborde-Payan-Xuong Conjecture.

2010 Mathematics Subject Classification: 05C20.

REFERENCES

- [1] J. Bang-Jensen and G. Gutin, *Digraphs. Theory, Algorithms and Applications* (Springer-Verlag Berlin Heidelberg New York, 2002).
- [2] V. Chvátal, On the computational complexity of finding a kernel, Technical Report Centre de Recherches Mathématiques, Université de Montréal **CRM-300** (1973).
- [3] H. Galeana-Sánchez and R. Gómez, *Independent sets and non-augmentable paths in generalizations of tournaments*, Discrete Math. **308** (2008) 2460–2472.
doi:10.1016/j.disc.2007.05.016
- [4] H. Galeana-Sánchez, R. Gómez and J.J. Montellano-Ballesteros, *Independent transversals of longest paths in locally semicomplete and locally transitive digraphs*, Discuss. Math. Graph Theory **29** (2009) 469–480.
doi:10.7151/dmgt.1458
- [5] H. Galeana-Sánchez and C. Hernández-Cruz, *On the existence of (k, l) -kernels in digraphs with a given circumference*, AKCE Int. J. Graphs Comb. **10** (2013) 15–28.
- [6] H. Galeana-Sánchez and C. Hernández-Cruz, *k -kernels in generalizations of transitive digraphs*, Discuss. Math. Graph Theory **31** (2011) 293–312.
doi:10.7151/dmgt.1546
- [7] H. Galeana-Sánchez and C. Hernández-Cruz, *On the existence of (k, l) -kernels in infinite digraphs: A survey*, Discuss. Math. Graph Theory **34** (2011) 431–466.
doi:10.7151/dmgt.1747
- [8] H. Galeana-Sánchez and C. Hernández-Cruz, *Cyclically k -partite digraphs and k -kernels*, Discuss. Math. Graph Theory **31** (2011) 63–79.
doi:10.7151/dmgt.1530
- [9] H. Galeana-Sánchez and C. Hernández-Cruz, *k -kernels in k -transitive and k -quasi-transitive digraphs*, Discrete Math. **312** (2012) 2522–2530.
doi:10.1016/j.disc.2012.05.005
- [10] P. Hell and C. Hernández-Cruz, *On the complexity of the 3-kernel problem in some classes of digraphs*, Discuss. Math. Graph Theory **34** (2014) 167–186.
doi:10.7151/dmgt.1727
- [11] C. Hernández-Cruz, *3-transitive digraphs*, Discuss. Math. Graph Theory **32** (2013) 205–219.
doi:10.7151/dmgt.1613
- [12] C. Hernández-Cruz, *4-transitive digraphs I: The structure of strong 4-transitive digraphs*, Discuss. Math. Graph Theory **33** (2013) 247–260.
doi:10.7151/dmgt.1645
- [13] C. Hernández-Cruz and J.J. Montellano-Ballesteros, *Some remarks on the structure of strong k -transitive digraphs*, Discuss. Math. Graph Theory **34** (2015) 651–672.
doi:10.7151/dmgt.1765
- [14] J.M. Laborde, C. Payan and N.H. Xuong, *Independent sets and longest paths in digraphs*, Graphs and Other Combinatorial Topics, Proceedings of the Third Czechoslovak Symposium of Graph Theory (1982) 173–177.

- [15] R. Wang, *$(k - 1)$ -kernels in strong k -transitive digraphs*, Discuss. Math. Graph Theory **35** (2015) 229–235.
doi:10.7151/dmgt.1787
- [16] S. Wang and R. Wang, *Independent sets and non-augmentable paths in arc-locally in-semicomplete digraphs and quasi-arc-transitive digraphs*, Discrete Math. **311** (2010) 282–288.
doi:10.1016/j.disc.2010.11.009

Received 14 February 2015
Revised 25 February 2016
Accepted 25 February 2016